

# KINEMATICS

## KINEMATICS

Study of motion of objects without taking into account the factors which cause the motion (i.e. nature of force).

### 1. FRAME OF REFERENCE

Motion of a body can be observed only if it changes its position with respect to some other body. Therefore, for motion to be observed there must be a body, which is changing its position with respect to another body and a person who is observing motion. The person observing motion is known as observer. The observer for the purpose of investigation must have its own clock to measure time and a point in the space attached with the other body as origin and a set of coordinate axes. These two things (the time measured by the clock and the coordinate system) are collectively known as reference frame.

In this way, motion of the moving body is expressed in terms of its position coordinates changing with time.

### 2. MOTION & REST

If a body changes its position with time, it is said to be moving otherwise it is at rest. Motion/rest is always relative to the observer.

Motion/rest is a combined property of the object under study and the observer. There is no meaning of rest or motion without the observer or frame of reference.

- To locate the position of a particle we need a reference frame. A commonly used reference frame is cartesian coordinate system or x-y-z coordinate system.

The coordinates (x, y, z) of the particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.

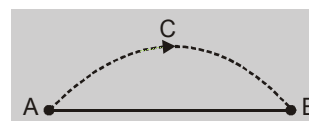
- If only one coordinate changes with time, motion is one dimensional (1 - D) or straight line motion.  
If only two coordinates change with time, motion is two dimensional (2 - D) or motion in a plane. If all three coordinates change with time, motion is three dimensional (3 - D) or motion in space.
- The reference frame is chosen according to problem.
- If frame is not mentioned, then ground is taken as reference frame.

### 3. DISTANCE & DISPLACEMENT

#### Distance

Distance is total length of path covered by the particle, in definite time interval.

Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.



But overall, body is displaced from A to B. A vector from A to B, i.e.  $\vec{AB}$  is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

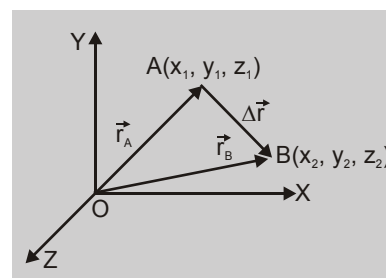
#### Displacement in terms of position vector

Let a body be displaced from  $A(x_1, y_1, z_1)$  to  $B(x_2, y_2, z_2)$  then its displacement is given by vector  $\vec{AB}$ .

$$\text{From } \triangle OAB \quad \vec{r}_A + \Delta\vec{r} = \vec{r}_B \quad \text{or} \quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\therefore \quad \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \quad \text{and} \quad \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\therefore \quad \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \quad \text{or} \quad \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$



## GOLDEN KEY POINTS

- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial positions.
- For a moving body, distance cannot have zero or negative values but displacement may be positive, negative or zero.
- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- Only single value of displacement is possible between two fixed points.
- If motion is in straight line without change in direction then  
 $\text{distance} = |\text{displacement}| = \text{magnitude of displacement}.$
- Magnitude of displacement may be equal or less than distance but never greater than distance.  
i.e.,  $\text{distance} \geq |\text{displacement}|$

## Illustrations

### Illustration 1.

A particle starts from the origin, goes along the X-axis upto the point (20m, 0) and then returns along the same line to the point (-20m, 0). Find the distance and displacement of the particle during the trip.

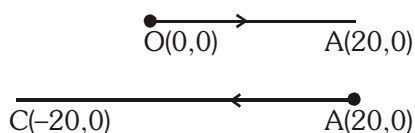
#### Solution

$$\text{Distance} = |OA| + |AC|$$

$$= 20 + 40 = 60\text{m}$$

$$\text{Displacement} = OA + AC$$

$$= 20\hat{i} + (-40\hat{i}) = (-20\hat{i})\text{m}$$

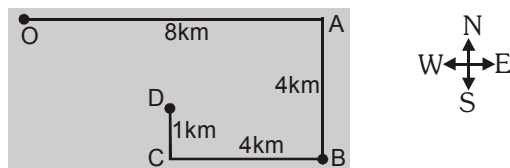


### Illustration 2.

A car moves from O to D along the path

OABCD shown in fig.

What is distance travelled and its net displacement ?



#### Solution

$$\text{Distance} = |\vec{OA}| + |\vec{AB}| + |\vec{BC}| + |\vec{CD}|$$

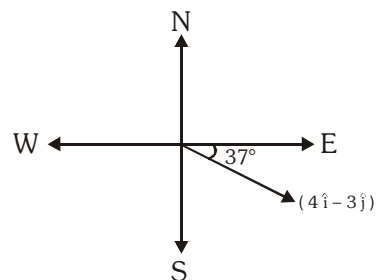
$$= 8 + 4 + 4 + 1 = 17 \text{ km}$$

$$\text{Displacement} = \vec{OA} + \vec{AB} + \vec{BC} + \vec{CD}$$

$$= 8\hat{i} + (-4\hat{j}) + (-4\hat{i}) + \hat{j} = 4\hat{i} - 3\hat{j}$$

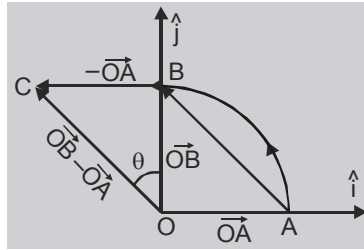
$$\Rightarrow |\text{displacement}| = \sqrt{(4)^2 + (3)^2} = 5$$

$$\text{So, Displacement} = 5 \text{ km, } 37^\circ \text{ S of E}$$



**Illustration 3.**

A particle goes along a quadrant of a circle of radius 10m from A to B as shown in fig. Find the magnitude of displacement and distance along the path AB, and angle between displacement vector and x-axis?

**Solution**

$$\text{Displacement } \vec{AB} = \vec{OB} - \vec{OA} = (10\hat{j} - 10\hat{i})\text{m}$$

$$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}\text{ m}$$

$$\text{From } \triangle OBC \tan\theta = \frac{OA}{OB} = \frac{10}{10} = 1 \Rightarrow \theta = 45^\circ$$

$$\text{Angle between displacement vector } \vec{OC} \text{ and x-axis} = 90^\circ + 45^\circ = 135^\circ$$

$$\text{Distance of path AB} = \frac{1}{4} (\text{circumference}) = \frac{1}{4} (2\pi R) \text{ m} = (5\pi) \text{ m}$$

**Illustration 4.**

On an open ground a motorist follows a track that turns to his left by an angle of  $60^\circ$  after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

**Solution**

At III turn

$$\begin{aligned} |\text{Displacement}| &= |\vec{OA} + \vec{AB} + \vec{BC}| = |\vec{OC}| \\ &= 500 \cos 60^\circ + 500 + 500 \cos 60^\circ \\ &= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2} = 1000 \text{ m} \end{aligned}$$

So  $|\text{Displacement}| = 1000 \text{ m}$  from O to C

$$\text{Distance} = 500 + 500 + 500 = 1500 \text{ m} \quad \therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{1000}{1500} = \frac{2}{3}$$

At VI turn

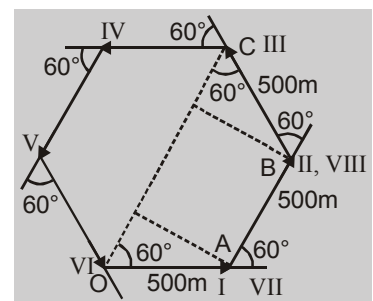
$\therefore$  initial and final positions are same so  $|\text{displacement}| = 0$  and distance  $= 500 \times 6 = 3000 \text{ m}$

$$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{0}{3000} = 0$$

At VIII turn

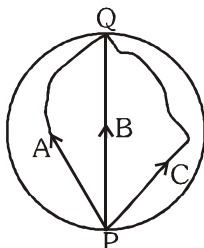
$$|\text{Displacement}| = 2(500)\cos\left(\frac{60^\circ}{2}\right) = 1000 \times \cos 30^\circ = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} \text{ m}$$

$$\text{Distance} = 500 \times 8 = 4000 \text{ m} \quad \therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$$

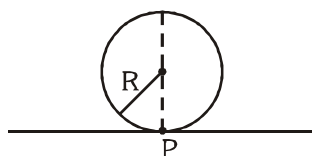


## BEGINNER'S BOX-1

1. A particle moves on a circular path of radius 'r'. It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s.
2. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement for each? For which girl is this equal to the actual length of path skate?



3. A wheel of radius 'R' is placed on ground and its contact point is 'P'. If wheel starts rolling without slipping and completes half a revolution, find the displacement of point P.



4. A man moves 4 m along east direction, then 3m along north direction, after that he climbs up a pole to a height 12m. Find the distance covered by him and his displacement.
5. A person moves on a semicircular track of radius 40 m. If he starts at one end of the track and reaches the other end, find the distance covered and magnitude of displacement of the person.
6. A man has to go 50m due north, 40m due east and 20m due south to reach a cafe from his home.  
(A) What distance he has to walk to reach the cafe? (B) What is his displacement from his home to the cafe?



## 4. SPEED & VELOCITY

### 4.1 Speed

The rate at which distance is covered with respect to time is called speed. It is a scalar quantity

Dimension :  $[M^0L^1T^{-1}]$

Unit : m/s (S.I.), cm/s (C.G.S.)

**Note :** For a moving particle speed can never be negative or zero, it is always positive.

#### Uniform speed

When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

$$\text{Uniform speed} = \frac{\text{Distance}}{\text{Time}}$$

#### Non-uniform (variable) speed

In non-uniform speed particle covers unequal distances in equal intervals of time.

**Average speed :** The average speed of a particle for a given 'interval of time' is defined as the ratio of total distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Time taken}} \quad \text{i.e. } v_{av} = \frac{\Delta s}{\Delta t}$$



## GOLDEN KEY POINTS

- When a particle moves with different uniform speeds  $v_1, v_2, v_3, \dots, v_n$  in different time intervals  $t_1, t_2, t_3, \dots, t_n$  respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

If  $t_1 = t_2 = t_3 = \dots = t_n$  then

$$v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n} \quad (\text{Arithmetic mean of speeds})$$

- When a particle describes different distances  $s_1, s_2, s_3, \dots, s_n$  with speeds  $v_1, v_2, v_3, \dots, v_n$  respectively then the average speed of particle over the total distance will be given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}$$

If  $s_1 = s_2 = s_3 = \dots = s_n$  then

$$v_{av} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}} \quad (\text{Harmonic mean of speeds})$$

### Instantaneous speed

It is the speed of a particle at a particular instant of time.

$$\text{Instantaneous speed } v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

## 4.2 Velocity

The rate of change of position *i.e.* rate of displacement with time is called velocity.

It is a vector quantity

Dimension :  $[M^0 L^1 T^{-1}]$

Unit : m/s (S.I.), cm/s (C.G.S.)

## GOLDEN KEY POINTS

- Velocity may be positive, negative or zero.
- Direction of velocity is always in the direction of change in position.
- Speedometer measures the instantaneous speed of a vehicle.

### Uniform velocity

A particle is said to have uniform velocity, if magnitude as well as direction of its velocity remain same. This is possible only when it moves in a straight line without reversing its direction.

### Non-uniform velocity

A particle is said to have non-uniform velocity, if both either magnitude or direction of velocity change.

### Average velocity

It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

Its direction is along the displacement.



## GOLDEN KEY POINTS

- If velocity is continuously changing with time i.e. velocity is the function of time then time average velocity

$$\langle v \rangle_t = \frac{\int v \, dt}{\int dt}$$

- If velocity is continuously changing with distance i.e. velocity is the function of space (distance) then space average velocity :-

$$\langle v \rangle_s = \frac{\int v \, ds}{\int ds}$$

- Average speed  $\geq$  | Average velocity |

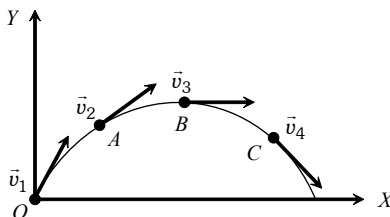
### Instantaneous velocity

It is the velocity of a particle at a particular instant of time.

$$\text{Instantaneous velocity } \vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

## GOLDEN KEY POINTS

- The direction of instantaneous velocity is always tangential to the path followed by the particle.



- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- A particle may have constant speed but variable velocity.

*Example :* When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.

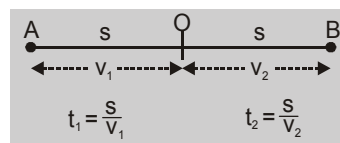
## Illustrations

### Illustration 5.

If a particle travels the first half distance with speed  $v_1$  and second half distance with speed  $v_2$ . Find its average speed during the journey.

#### Solution

$$v_{av} = \frac{s + s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$



**Note :-** Here  $v_{av}$  is the harmonic mean of two speeds.



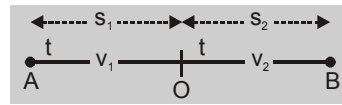
**Illustration 6.**

If a particle travels with speed  $v_1$  during first half time interval and with speed  $v_2$  during second half time interval. Find its average speed during its journey.

**Solution**

$$s_1 = v_1 t \text{ and } s_2 = v_2 t$$

$$\text{Total distance} = s_1 + s_2 = (v_1 + v_2)t$$



$$\text{total time} = t + t = 2t \quad \text{then} \quad v_{av} = \frac{s_1 + s_2}{t + t} = \frac{(v_1 + v_2)t}{2t} = \frac{v_1 + v_2}{2}$$

**Note :-** here  $v_{av}$  is arithmetic mean of two speeds.

**Illustration 7.**

A car moves with a velocity 2.24 km/h in first minute, with 3.60 km/h in the second minute and with 5.18 km/h in the third minute. Calculate the average velocity in these three minutes.

**Solution**

$$\text{Distance travelled in first minute} \quad s_1 = v_1 \times t_1 = 2.24 \times \frac{1}{60} \text{ km}$$

$$\text{Distance travelled in second minute} \quad s_2 = v_2 \times t_2 = 3.60 \times \frac{1}{60} \text{ km}$$

$$\text{Distance travelled in third minute} \quad s_3 = v_3 \times t_3 = 5.18 \times \frac{1}{60} \text{ km}$$

$$\text{Total distance travelled, } s = s_1 + s_2 + s_3 = \frac{2.24}{60} + \frac{3.60}{60} + \frac{5.18}{60} = \frac{11.02}{60} \text{ km}$$

$$\text{Total time taken, } t = 1 + 1 + 1 = 3 \text{ min} = \frac{1}{20} \text{ h}$$

$$\therefore \text{ average velocity} = \frac{s}{t} = \frac{11.02}{60} \times \frac{20}{1} = 3.67 \text{ km/h}$$

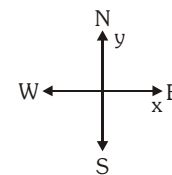
**Illustration 8.**

A bird flies due north with velocity 20 m/s for 15 s it rests for 5 s and then flies due south with velocity 24 m/s for 10 s. Find the average speed and magnitude of average velocity. For the whole trip.

**Solution**

$$\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{20 \times 15 + 24 \times 10}{15 + 5 + 10} = \frac{540}{30} = 18 \text{ m/s}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Total Time}} = \frac{(20 \times 15)\hat{j} + (24 \times 10)(-\hat{j})}{15 + 5 + 10} = \frac{60\hat{j}}{30} = 2\hat{j}$$



$$\text{Magnitude of average velocity} = |2\hat{j}| = 2 \text{ m/s}$$

**Illustration 9.**

The displacement of a point moving along a straight line is given by

$$s = 4t^2 + 5t - 6$$

Here  $s$  is in cm and  $t$  is in seconds calculate

(i) Initial speed of particle

(ii) Speed at  $t = 4s$

**Solution**

$$(i) \text{ Speed, } v = \frac{ds}{dt} = 8t + 5 \quad \text{Initial speed (i.e at } t = 0), v = 5 \text{ cm/s}$$

$$(ii) \text{ At } t = 4s, v = 8(4) + 5 = 37 \text{ cm/s}$$



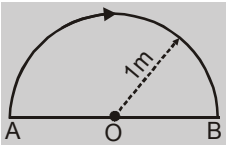
### Illustration 10.

- (a) If  $s = 2t^3 + 3t^2 + 2t + 8$  then find time at which acceleration is zero.  
(b) Velocity of a particle (starting at  $t = 0$ ) varies with time as  $v = 4t$ . Calculate the displacement of particle between  $t = 2$  to  $t = 4$  s [AIPMT Mains - 2004]

### Solution

- (a)  $v = \frac{ds}{dt} = 6t^2 + 6t + 2 \Rightarrow a = \frac{dv}{dt} = 12t + 6 = 0 \Rightarrow t = -\frac{1}{2}$  which is impossible. Therefore acceleration can never be zero.  
(b)  $\therefore \frac{dx}{dt} = v \therefore x = \int v dt = \int_2^4 4t dt = \left[ 2t^2 \right]_2^4 = 2(4)^2 - 2(2)^2 = 32 - 8 = 24$  m

## BEGINNER'S BOX-2

- Air distance between Kota to Jaipur is 260 km and road distance is 320 km. A deluxe bus which moves from Jaipur to Kota takes 8 h while an aeroplane reaches in just 15 min. Find
  - average speed of bus in km/h
  - average velocity of bus in km/h
  - average speed of aeroplane in km/h
  - average velocity of aeroplane in km/h
- A particle moves on a straight line in such way that it covers 1st half distance with speed 3 m/s and next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find average speed of the particle.
- Length of a minute hand of a clock is 4.5 cm. Find the average velocity of the tip of minute's hand between 6 A.M. to 6.30 A.M. & 6 A.M. to 6.30 P.M.
- A particle of mass 2 kg moves on a circular path with constant speed 10 m/s. Find change in speed and magnitude of change in velocity. When particle completes half revolution.
- The distance travelled by a particle in time  $t$  is given by  $s = (2.5 t^2)$  m. Find (a) the average speed of the particle during time 0 to 5.0s and (b) the instantaneous speed at  $t = 5.0$  s.
- A particle goes from point A to point B, moving in a semicircle of radius 1m in 1 second. Find the magnitude of its average velocity.
- Straight distance between a hotel and a railway station is 10 km, but circular route is followed by a taxi covering 23 km in 28 minute. What is average speed and magnitude of average velocity? Are they equal ?

## 5. ACCELERATION

The rate of change of velocity of an object is called acceleration of the object.

It is a vector quantity. It's direction is same as that of change in velocity (Not in the direction of the velocity).

Dimension :  $[M^0L^1T^{-2}]$

Unit :  $m/s^2$  (S.I.);  $cm/s^2$  (C.G.S.)

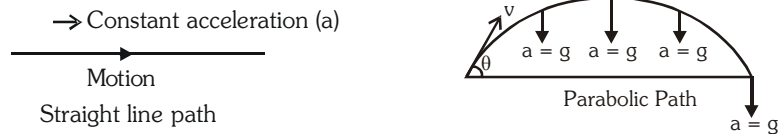
### Uniform acceleration

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle .



## GOLDEN KEY POINTS

- When a particle moves with constant acceleration, then its path may be straight line or parabolic.



- When a particle starts from rest and moves with constant acceleration then its path must be a straight line.



- When a particle moves with variable velocity then acceleration must be present.
- When a particle moves continuously on a same straight line with uniform speed then acceleration of the particle is zero.
- When a particle moves continuously on a curved path with uniform speed then acceleration of the particle is non zero. For example uniform circular motion is an accelerated motion
- For a particle moving with uniform velocity acceleration must be zero.

### Non-uniform acceleration

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

### Average acceleration

It is the ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

### Instantaneous acceleration

It is the acceleration of a particle at a particular instant of time.

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

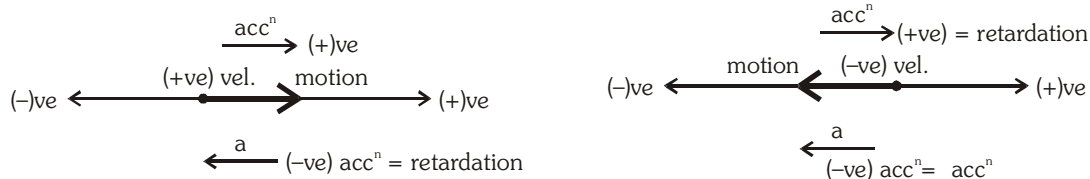
i.e. first derivative of velocity is called instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \quad \left[ \text{As } \vec{v} = \frac{d\vec{r}}{dt} \right]$$

i.e. second derivative of position vector is called instantaneous acceleration

## GOLDEN KEY POINTS

- When a particle moves with non-uniform speed then acceleration of the particle must be non zero.
- The direction of average acceleration vector is the direction of the change in velocity vector as  $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$
- Acceleration which opposes the motion of body is called retardation.



- Sign of velocity(+ve or -ve) represents the direction of motion but sign of acceleration indicates the direction of acceleration
- If velocity and acceleration both are having same sign, then magnitude of velocity (i.e speed) is increasing and if both have opposite signs, then magnitude of velocity (i.e. speed) is decreasing.



## Illustrations

### Illustration 11.

The velocity of a particle is given by  $v = (2t^2 - 4t + 3)$  m/s where  $t$  is time in seconds. Find its acceleration at  $t = 2$  second.

#### Solution

$$\text{Acceleration (a)} = \frac{dv}{dt} = \frac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$$

Therefore acceleration at  $t = 2$  s is equal to,  $a = (4 \times 2) - 4 = 4 \text{ m/s}^2$

### Illustration 12.

The velocity of particle moving in the positive direction of  $x$  axis varies as  $v = \alpha\sqrt{x}$ , where  $\alpha$  is a positive constant. Assuming that at moment  $t=0$ , the particle was located at the point  $x=0$ . Find

- the time dependence of the velocity and the acceleration of the particle.
- the mean velocity of the particle averaged over the time that the particle takes to cover first  $s$  meters of the path.

#### Solution

$$(a) \quad v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \alpha \int_0^t dt \Rightarrow \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \alpha t \Rightarrow x = \frac{\alpha^2 t^2}{4}$$

$$\text{Velocity } v = \frac{dx}{dt} = \frac{2\alpha^2 t}{4} = \frac{1}{2}\alpha^2 t$$

$$\text{Acceleration } a = \frac{dv}{dt} = \frac{\alpha^2}{2}$$

- Time taken to cover the first  $s$  distance

$$x = \frac{\alpha^2 t^2}{4} \Rightarrow s = \frac{\alpha^2 t^2}{4} \Rightarrow t = \sqrt{\frac{4s}{\alpha^2}} \Rightarrow v_{av} = \frac{s}{t} = \frac{s}{\sqrt{\frac{4s}{\alpha^2}}} = \frac{\alpha\sqrt{s}}{2}$$

### Illustration 13.

A particle is moving along a straight line OX. At a time  $t$  (in seconds) the distance  $x$  (in metres) of particle from point O is given by  $x = 10 + 6t - 3t^2$ . How long would the particle travel before coming to rest?

#### Solution

Initial value of  $x$ , at  $t = 0$ ,  $x_1 = 10$  m

$$\text{Velocity } v = \frac{dx}{dt} = 6 - 6t \quad \text{When } v = 0, \quad t = 1 \text{ s}$$

Final value of  $x$ , at  $t = 1$  s,  $x_2 = 10 + 6 \times 1 - 3(1^2) = 13$  m

Distance travelled =  $x_2 - x_1 = 13 - 10 = 3$  m

### Illustration 14.

The acceleration of a particle moving in a straight line varies with its displacement as,  $a = 2s+1$  velocity of the particle is zero at zero displacement. Find the corresponding velocity - displacement equation.

#### Solution

$$a = 2s + 1 \Rightarrow \frac{dv}{dt} = 2s + 1 \Rightarrow \frac{dv}{ds} \cdot \frac{ds}{dt} = 2s + 1 \Rightarrow \frac{dv}{ds} \cdot v = 2s + 1$$

$$\Rightarrow \int_0^v v dv = 2 \int_0^s s ds + \int_0^s ds$$

$$\Rightarrow \left( \frac{v^2}{2} \right)_0^v = 2 \left( \frac{s^2}{2} \right)_0^s + [s]_0^s \Rightarrow \frac{v^2}{2} = s^2 + s \Rightarrow v = \sqrt{2s(s+1)}$$



### BEGINNER'S BOX-3

1. A particle moves on circular path of radius 5 m with constant speed 5 m/s. Find the magnitude of its average acceleration when it completes half revolution.
2. The position of a particle moving on X-axis is given by  $x = At^2 + Bt + C$ . The numerical values of A, B and C are 7, -2 and 5 respectively and SI units are used. Find
  - (a) The velocity of the particle at  $t = 5$
  - (b) The acceleration of the particle at  $t = 5$
  - (c) The average velocity during the interval  $t = 0$  to  $t = 5$
  - (d) The average acceleration during the interval  $t = 0$  to  $t = 5$

## 6. EQUATIONS OF MOTION

Equations of motion are valid when acceleration is constant.

- $v = u + at$

- $s = ut + \frac{1}{2}at^2$

- $v^2 - u^2 = 2as$

- $s_{n^{\text{th}}} = u + \frac{1}{2}a(2n - 1)$

- $s = v_{av}t = \frac{(u + v)}{2}t$

- $s = vt - \frac{1}{2}at^2$

$a$	= acceleration = constant
$u$	= Initial velocity
$v$	= Final velocity
$s$	= Displacement
$s_{n^{\text{th}}}$	= Displacement in the $n^{\text{th}}$ second

## Illustrations

### Illustration 15.

For a particle moving with constant acceleration, prove that the displacement in the  $n^{\text{th}}$  second is given by

$$s_{n^{\text{th}}} = u + \frac{a}{2}(2n - 1).$$

### Solution

From  $s = ut + \frac{1}{2}at^2$

$$s_n = un + \frac{1}{2}an^2 \dots\dots (1)$$

$$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \dots\dots (2)$$

By equation (1) & (2)

$$s_n - s_{n-1} = s_{n^{\text{th}}} = u + \frac{a}{2}(2n - 1)$$



## GOLDEN KEY POINTS

- Identification of equation of motion
  - (i) If  $t = \text{given}$  and  $v = ?$  then use  
 $v = u + at$
  - (ii) If  $t = \text{given}$  and  $s = ?$  then use  
 $s = ut + \frac{1}{2}at^2$
  - (iii) If  $s = \text{given}$  and  $v = ?$  then use  
 $v^2 = u^2 + 2as$
- All the equations of motion can be used in 2-D motion in vector form.

### Vector Form of Equations of motion

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2\vec{a} \cdot \vec{s}$$

$$\vec{s}_{n^{\text{th}}} = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$$

$$\vec{s} = \left( \frac{\vec{u} + \vec{v}}{2} \right) t$$

$$\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

### Concept of stopping distance and stopping time

A body moving with a velocity  $u$  is stopped by application of brakes after covering a distance  $s$ . If the same body moves with velocity  $nu$  and same braking force is applied on it then it will come to rest after covering a distance of  $n^2s$ .

$$\text{As } v^2 = u^2 - 2as$$

$$\Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a} \Rightarrow s \propto u^2 \quad [\text{since } a \text{ is constant}]$$

So we can say that if  $u$  becomes  $n$  times then  $s$  becomes  $n^2$  times that of previous value.

### Stopping time :

$$v = u - at$$

$$\Rightarrow 0 = u - at$$

$$\Rightarrow t = \frac{u}{a} \Rightarrow t \propto u \quad [\text{since } a \text{ is constant}]$$

So we can say that if  $u$  becomes  $n$  times then  $t$  becomes  $n$  times that of previous value.

## Illustrations

### Illustration 16.

Two cars start off a race with velocities  $2\text{m/s}$  and  $4\text{m/s}$  travel in straight line with uniform accelerations  $2\text{m/s}^2$  and  $1\text{m/s}^2$  respectively. What is the length of the path if they reach the final point at the same time ?  
[AIPMT (Main) -2008]

### Solution

Let both particles reach at same position in same time  $t$  then from  $s = ut + \frac{1}{2}at^2$

$$\text{For 1st particle : } s = 4(t) + \frac{1}{2}(1)t^2 = 4t + \frac{t^2}{2}, \text{ For 2nd particle : } s = 2(t) + \frac{1}{2}(2)t^2 = 2t + t^2$$

$$\text{Equating above equation we get } 4t + \frac{t^2}{2} = 2t + t^2 \Rightarrow t = 4 \text{ s}$$

$$\text{Substituting value of } t \text{ in above equation } s = 4(4) + \frac{1}{2}(1)(4)^2 = 16 + 8 = 24 \text{ m}$$

### Illustration 17.

A particle moves in a straight line with a uniform acceleration  $a$ . Initial velocity of the particle is zero. Find the average velocity of the particle in first ' $s$ ' distance.

### Solution

$$\therefore s = \frac{1}{2}at^2 \quad \therefore \frac{s^2}{t^2} = \frac{1}{2}as$$

$$\text{Average velocity} = \frac{s}{t} = \sqrt{\frac{as}{2}}$$



### Illustration 18.

A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops ?

### Solution

For retardation of train  $v^2 = u^2 + 2as \Rightarrow 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100 \text{ km/hr}^2$

Time required to stop the train  $v = u + at \Rightarrow 0 = 20 - 100t \Rightarrow t = \frac{1}{5} \text{ hr}$

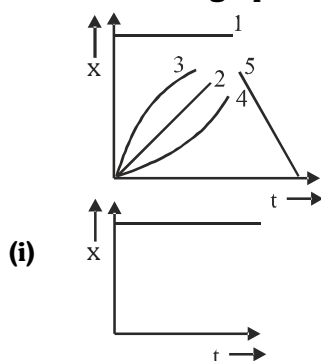
For Bird, speed =  $\frac{\text{Distance}}{\text{time}} \Rightarrow s_B = v_B \times t = 60 \times \frac{1}{5} = 12 \text{ km.}$

## BEGINNER'S BOX-4

1. A particle starts from rest, moves with constant acceleration for 15s. If it covers  $s_1$  distance in first 5s then distance  $s_2$  in next 10s, then find the relation between  $s_1$  &  $s_2$ .
2. The engine of a train passes an electric pole with a velocity 'u' and the last compartment of the train crosses the same pole with a velocity v. Then find the velocity with which the mid-point of the train passes the pole. Assume acceleration to be uniform.
3. A bullet losses  $1/n$  of its velocity in passing through a plank. What is the least number of planks required to stop the bullet ? (Assuming constant retardation)
4. A car moving along a straight highway with speed  $126 \text{ km h}^{-1}$  is brought to a halt within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?
5. A car is moving with speed u. Driver of the car sees red traffic light. His reaction time is t, then find out the distance travelled by the car after the instant when the driver decided to apply brakes. Assume uniform retardation 'a' after applying brakes.
6. If a body starts from rest and travels 120cm in the 6<sup>th</sup> second then what is the acceleration ?

## 7. GRAPHICAL SECTION

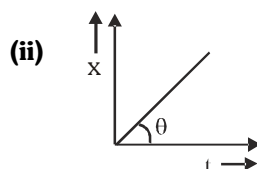
### Position - time graph



$\theta = 0^\circ$   
 $\tan \theta = \tan 0^\circ = 0$   
velocity = 0  
i.e. body is at rest.

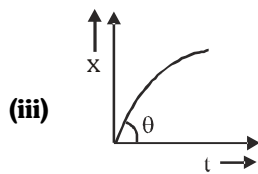
Slope of this graph represents instantaneous velocity.

$$\therefore \tan \theta = \frac{\text{displacement}}{\text{time}} = \text{velocity}$$

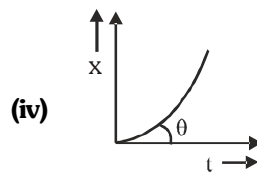


$\theta = \text{constant}$   
 $\tan \theta = \text{constant}$   
velocity = constant  
i.e. the body is in uniform motion

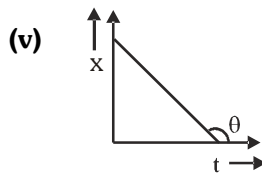




$\theta$  is decreasing with time  
 $\therefore \tan\theta$  is decreasing with time  
 $\therefore$  velocity is decreasing with time  
 i.e. non uniform motion



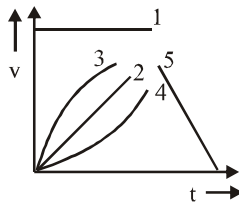
$\theta$  is increasing with time  
 $\therefore \tan\theta$  is increasing with time  
 $\therefore$  velocity is increasing with time  
 i.e. non uniform motion



$\theta > 90^\circ$   
 $\tan\theta = -ve$   
 velocity = -ve but constant  
 i.e. uniform motion

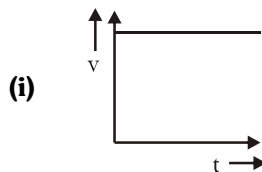
Area of x-t graph =  $\int x dt$  = No physical significance

### Velocity time graph

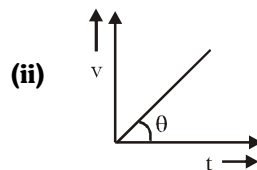


Slope of this graph represents acceleration.

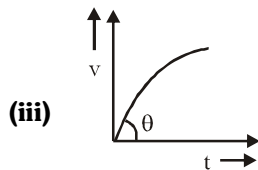
$$\therefore \tan\theta = \frac{\text{velocity}}{\text{time}} = \text{acceleration}$$



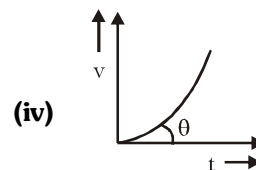
$\theta = 0^\circ$   
 $\tan\theta = \tan 0^\circ = 0$   
 acceleration = 0  
 i.e.  $v = \text{constant}$  or uniform motion



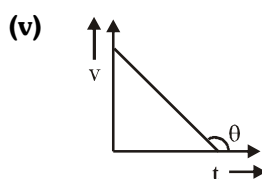
$\theta = \text{constant}$   
 $\tan\theta = \text{constant}$   
 acceleration = constant  
 i.e. uniformly accelerated motion



$\theta$  is decreasing with time  
 $\therefore \tan\theta$  is decreasing with time  
 $\therefore$  acceleration is decreasing with time  
 i.e. acceleration goes on decreasing with time but it is not retardation



$\theta$  is increasing with time  
 $\therefore \tan\theta$  is increasing with time  
 $\therefore$  acceleration is increasing with time  
 i.e. acceleration goes on increasing with time



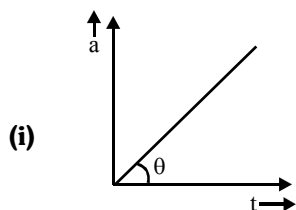
$\theta > 90^\circ$   
 $\tan\theta = -ve$   
 acceleration = -ve but constant  
 i.e. constant or uniform retardation is acting on the body

Area of v-t graph =  $\int v dt$  = displacement = change in position

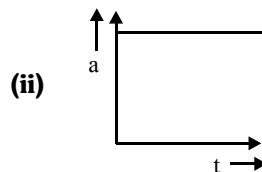


### Acceleration-time graph

$$\text{Area of } a-t \text{ graph} = \int a \, dt = \int dv = v_2 - v_1 = \text{change in velocity}$$



i.e. uniformly increasing acceleration.

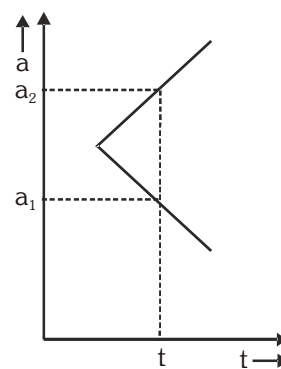
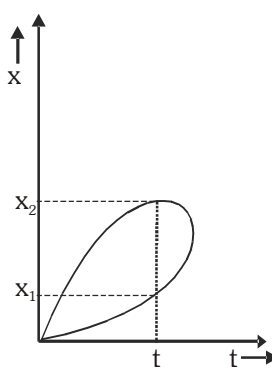
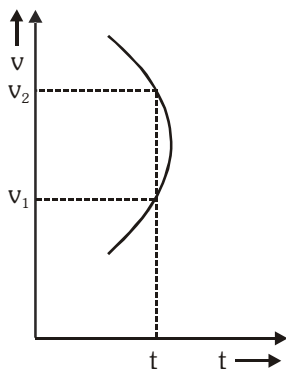


$a \propto t^0$  i.e. uniform or constant acceleration

### GOLDEN KEY POINTS

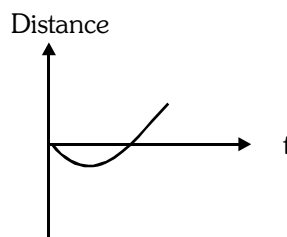
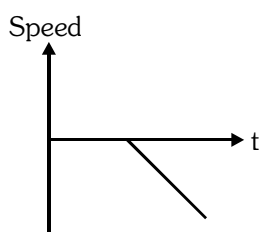
- Total area enclosed between speed-time (v-t) graph and time axis represent distance.
- Vector sum of total area enclosed between v-t graph and time axis represent displacement.
- Following graphs do not exist in practice :

#### Case-I



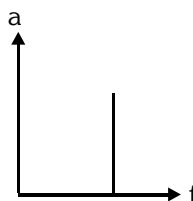
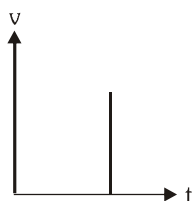
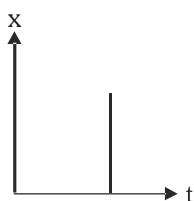
**Explanation :** In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

#### Case- II



**Explanation :** Speed or distance can never be negative.

#### Case - III



**Explanation :** It is not possible to change any quantity without consuming time i.e. time can't be constant.



## Illustrations

### Illustration 19.

A car starting from rest, accelerates at the rate  $f$  through a distance  $S$ , then continues at constant speed for time  $t$  and then comes to rest with retardation  $\frac{f}{2}$ . If the total distance travelled is  $15S$  then calculate the value of  $S$  in term of  $f$  and  $t$ .

### Solution

Let constant speed be  $v_m$

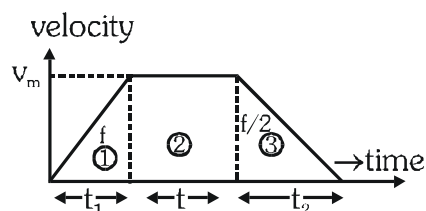
for time  $t_1$ ;  $v_m = ft_1$  and  $S = \frac{1}{2} ft_1^2$

for time  $t_2$   $0 = v_m - \frac{f}{2}t_2 \Rightarrow t_2 = 2t_1$

$$S_3 = \frac{1}{2} \left( \frac{f}{2} \right) t_2^2 = \left( \frac{f}{4} \right) (4t_1^2) = ft_1^2 = 2S$$

Therefore  $S + v_m t + 2S = 15S \Rightarrow v_m t = 12S \Rightarrow ft_1 t = 12S$

$$\Rightarrow f \left( \frac{2S}{f} \right)^{\frac{1}{2}} t = 12S \Rightarrow 2Sf = \frac{144S^2}{t^2} \Rightarrow S = \frac{ft^2}{72}$$



### Illustration 20.

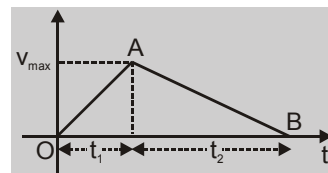
A car accelerates from rest at a constant rate  $\alpha$  for some time, after which it decelerates at a constant rate  $\beta$ , to come to rest. If the total time elapsed is  $t$ , evaluate (a) the maximum velocity attained (b) the total distance travelled.

### Solution

- (a) Let the car accelerate for time  $t_1$  and decelerate for time  $t_2$  then

$$t = t_1 + t_2 \quad \dots(i)$$

and corresponding velocity-time graph will be as shown in fig.



From the graph  $\alpha = \text{slope of line OA} = \frac{v_{\max}}{t_1}$  or  $t_1 = \frac{v_{\max}}{\alpha} \quad \dots(ii)$

and  $\beta = -\text{slope of line AB} = \frac{v_{\max}}{t_2}$  or  $t_2 = \frac{v_{\max}}{\beta} \quad \dots(iii)$

From Eqs. (i), (ii) and (iii)  $\frac{v_{\max}}{\alpha} + \frac{v_{\max}}{\beta} = t$  or  $v_{\max} \left( \frac{\alpha + \beta}{\alpha\beta} \right) = t$

$$\text{or } v_{\max} = \frac{\alpha\beta t}{\alpha + \beta}$$

(b) Total distance = area under  $v$ - $t$  graph  $= \frac{1}{2} \times t \times v_{\max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha + \beta}$

$$\text{Distance} = \frac{1}{2} \left( \frac{\alpha\beta t^2}{\alpha + \beta} \right).$$

**Note:** This problem can also be solved by using equations of motion ( $v = u + at$ , etc.).



**Illustration 21.**

A rocket is fired upwards vertically with a net acceleration of  $4 \text{ m/s}^2$  and initial velocity zero. After 5 seconds its fuel is finished and it retards with  $g$ . At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration  $g$  and returns back to ground. Plot the velocity-time and displacement-time graphs for the complete journey. Take  $g = 10 \text{ m/s}^2$ .

**Solution**

In the graphs,  $v_A = at_{OA} = (4)(5) = 20 \text{ m/s}$ ,

$$v_B = 0 = v_A - gt_{AB}$$

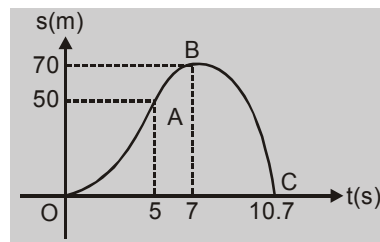
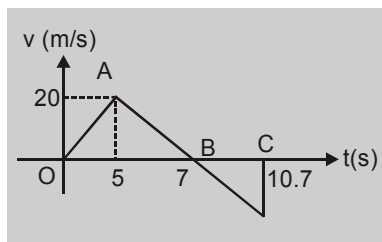
$$\therefore t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2\text{s} \quad \therefore t_{OAB} = (5+2)\text{s} = 7\text{s}$$

$$\text{Now, } s_{OAB} = \text{area under } v\text{-}t \text{ graph between } 0 \text{ to } 7 \text{ s} = \frac{1}{2}(7)(20) = 70 \text{ m}$$

$$\text{Now, } s_{OAB} = s_{BC} = \frac{1}{2}gt_{BC}^2 \quad \therefore 70 = \frac{1}{2}(10)t_{BC}^2$$

$$\therefore t_{BC} = \sqrt{14} = 3.7\text{s} \quad \therefore t_{OAB} = 7+3.7 = 10.7\text{s}$$

$$\text{Also } s_{OA} = \text{area under } v\text{-}t \text{ graph between } OA = \frac{1}{2}(5)(20) = 50 \text{ m}$$

**Illustration 22.**

Velocity-time graph of a particle moving in a straight line is shown. Plot the corresponding displacement-time graph of the particle.

**Solution**

Displacement = area under velocity-time graph.

$$\text{Hence, } s_{OA} = \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$$

$$s_{AB} = 2 \times 10 = 20 \text{ m}$$

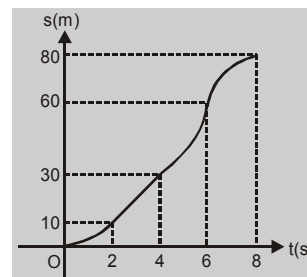
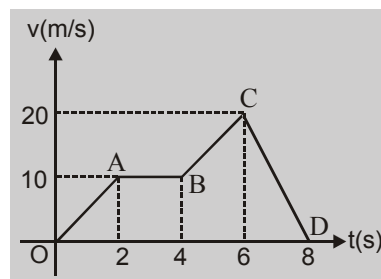
$$\text{or } s_{OAB} = 10+20 = 30 \text{ m}$$

$$s_{BC} = 2 \times \left( \frac{10+20}{2} \right) = 30 \text{ m}$$

$$\text{or } s_{OABC} = 30+30 = 60 \text{ m}$$

$$\text{and } s_{CD} = \frac{1}{2}(2 \times 20) = 20 \text{ m}$$

$$\text{or } s_{OABCD} = 60 + 20 = 80 \text{ m}$$



Between 0 to 2 s and 4 to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape. At the end of 8 s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in the figure.



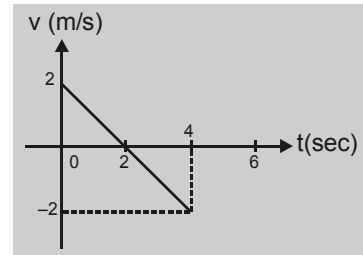
**Illustration 23.**

Velocity-time graph for a particle moving in a straight line is given  
Calculate the displacement of the particle and distance travelled in first 4 seconds.

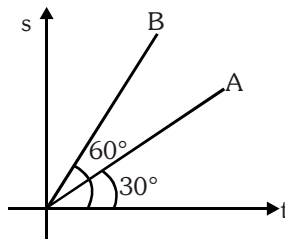
**Solution**

Take the area above time axis as positive and area below time axis negative then displacement  $= (2 - 2)\text{m} = 0$

while for distance take all areas as positive the distance covered  $s = (2 + 2)\text{m} = 4\text{m}$

**BEGINNER'S BOX-5**

1. s-t graph of two particles A and B are shown in fig. Find the ratio of velocity of A to velocity of B.

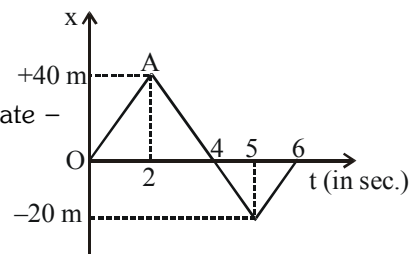


2. Position-time graph of a particle in motion is shown in fig. Calculate –

(i) Total distance covered

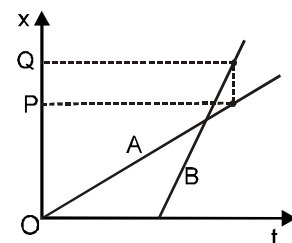
(ii) Displacement

(iii) Average speed (iv) Average velocity.



3. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose correct entries in the brackets below :

- (A/B) lives closer to the school than (B/A)
- (A/B) starts from the school earlier than (B/A)
- (A/B) walks faster than (B/A)
- A and B reach home at the (same / different) time
- (A/B) overtakes (B/A) on the road (once/ twice).



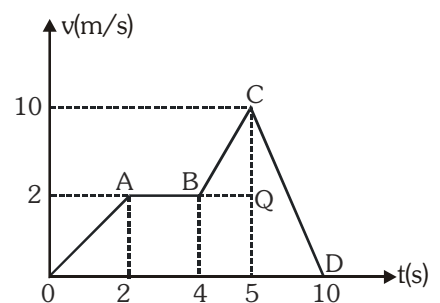
4. A particle moves on straight line according to the velocity-time graph shown in fig. Calculate –

(i) Total distance covered

(ii) Average speed

(iii) In which part of the graph the acceleration is maximum and also find its value.

(iv) Retardation



5. A body starts from rest and moves with a uniform acceleration of  $10 \text{ ms}^{-2}$  for 5 seconds. During the next 10 seconds it moves with uniform velocity. Find the total distance travelled by the body (Using graphical analysis).



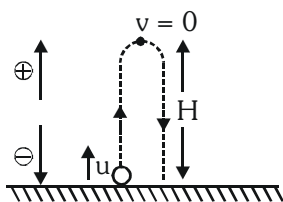
## 8. MOTION UNDER GRAVITY (FREE FALL)

Acceleration produced in a body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol  $g$ .

Value of  $g = 9.8 \text{ m/s}^2 = 980 \text{ cm/s}^2 = 32 \text{ ft/s}^2$

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude ( $h \ll \text{earth's radius}$ ) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

### 8.1 If a Body is Projected Vertically Upward



Positive / Negative directions are a matter of choice. You may take another choice.

- (i) **Equations of motion :** Taking initial position as origin and direction of motion (i.e. vertically up) as positive  $a = -g$  [As acceleration is downwards while motion upwards]  
So, if a body is projected with velocity  $u$  and after time  $t$  it reaches a height  $h$  then

$$v = u - gt, \quad h = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gh, \quad h_{n^{\text{th}}} = u - \frac{g}{2}(2n - 1)$$

- (ii) **For maximum height**  $v = 0$

So from above equation  $u = g t$

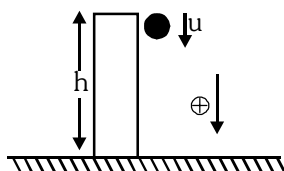
it is called time of ascent  $(t_1) = u/g$

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent  $(t_2) = \text{time of ascent } (t_1) = u/g$

$$\therefore \text{Total time of flight } T = t_1 + t_2 = \frac{2u}{g}$$

$$\text{and } u^2 = 2gH \Rightarrow H = \frac{u^2}{2g}$$

### 8.2 If a Body is Projected Vertically Downward With Some Initial Velocity From Some Height



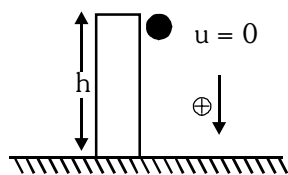
**Equations of motion :** Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, we have

$$v = u + gt \qquad h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh \qquad h_n = u + \frac{g}{2}(2n - 1)$$



### 8.3 If a body is dropped from some height (initial velocity zero)



**Equations of motion :** Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

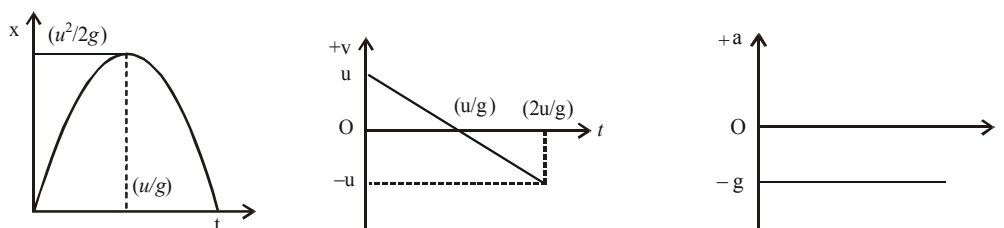
$$u = 0 \quad [\text{As body starts from rest}]$$

$$a = +g \quad [\text{As acceleration is in the direction of motion}]$$

$$\text{so } v = gt, \quad h = \frac{1}{2}gt^2$$

#### GOLDEN KEY POINTS

- In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- The magnitude of velocity at any point on the path is same whether the body is moving in upward or downward direction.
- Graph of displacement, velocity and acceleration with respect to time :  
(For a body projected vertically upward)



- As  $h = (1/2)gt^2$ , *i.e.*,  $h \propto t^2$ , distance covered in time  $t$ ,  $2t$ ,  $3t$ , etc., will be in the ratio of  $1^2 : 2^2 : 3^2$ , *i.e.*, square of consecutive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers *i.e.*

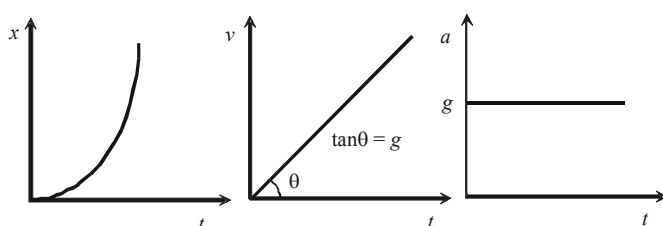
$$\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), (\sqrt{4} - \sqrt{3}), \dots$$

- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*,  $t = \sqrt{(2h/g)}$  and  $v = \sqrt{2gh}$ .

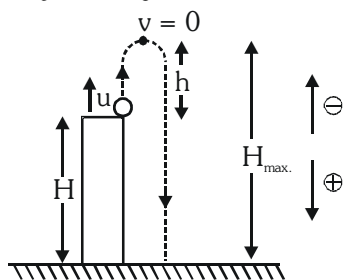
- The distance covered in the  $n^{\text{th}}$  second,  $h_n = \frac{1}{2}g(2n - 1)$

So distance covered in 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> second, etc., will be in the ratio of  $1 : 3 : 5$ , *i.e.*, odd integers only.

- Graph of distance, velocity and acceleration with respect to time :  
(For a body dropped from some height)



### 8.4. If a Body is Projected Vertically Upward With Some Initial Velocity From a Certain Height



**Equations of motion :** Taking initial position as origin and direction of motion (*i.e.*, upward direction) as negative, here we have

$$v = -u + gt; \quad H = -ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh; \quad h_{n^{th}} = -u + \frac{g}{2}(2n - 1)$$

- Maximum height attained by the body

$$H_{max} = H + h = H + \frac{u^2}{2g}$$

- Distance travelled by the body

$$H + 2h = H + \frac{u^2}{g}$$

Time taken by the body to reach the ground

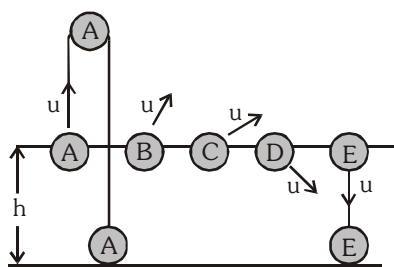
$$H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut - H = 0$$

$$\Rightarrow gt^2 - 2ut - 2H = 0$$

After solving this equation we get the result.

### GOLDEN KEY POINTS

- If various particles thrown with same initial speed but in different directions then

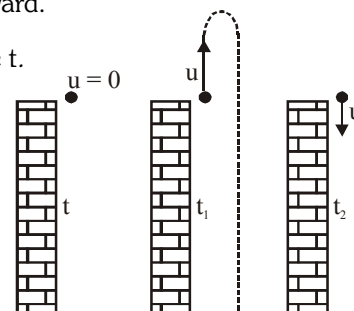


- They strike the ground with same speed at different times irrespective of their initial direction of velocities.
- Time would be least for particle E which was thrown vertically downward.
- Time would be maximum for particle A which was thrown vertically upward.

- A ball is dropped from a building of height h and it reaches ground after time t.

From the same building if two balls are thrown (one upwards and other downwards) with the same speed u and they reach the ground after  $t_1$  and  $t_2$  seconds respectively then

$$t = \sqrt{t_1 t_2}$$



### 8.5 A body is thrown vertically upwards, if Constant Air resistance is to be taken into account:

For upward motion :-

Net acceleration  $a_{\text{Net}} = g + a$  (downwards)

If maximum height attained by the particle is 'H' then

$$t_{\text{ascent}} = \sqrt{\frac{2H}{a_{\text{Net}}}}$$

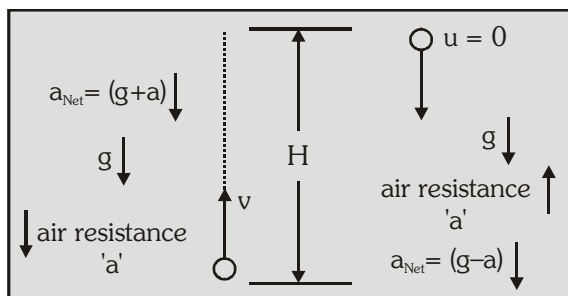
$$\Rightarrow t_{\text{ascent}} = \sqrt{\frac{2H}{g+a}}$$

For downward motion :-

Net acceleration  $a_{\text{Net}} = g - a$  (downwards)

$$\text{So } t_{\text{descent}} = \sqrt{\frac{2H}{g-a}}$$

Also  $t_{\text{descent}} > t_{\text{ascent}}$



### GOLDEN KEY POINTS

- For downward motion  $a$  and  $g$  will work in opposite directions because  $a$  always acts in direction opposite to motion and  $g$  always acts vertically downwards.

## Illustrations

### Illustration 24.

A body is dropped from a height  $h$  above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in 1<sup>st</sup> second, in 2<sup>nd</sup> second, in 3<sup>rd</sup> second etc.

### Solution

From second equation of motion, i.e.  $h = \frac{1}{2}gt^2$  ( $h = ut + \frac{1}{2}gt^2$  and  $u = 0$ )

$$h_1 : h_2 : h_3 : \dots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 : \dots = 1 : 4 : 9 : \dots$$

Now from the expression of distance travelled in  $n^{\text{th}}$  second  $S_n = u + \frac{1}{2}a(2n-1)$

here  $u = 0$ ,  $a = g$  So  $S_n = \frac{1}{2}g(2n-1)$  therefore

$$S_1 : S_2 : S_3 : \dots = \frac{1}{2}g(2 \times 1 - 1) : \frac{1}{2}g(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1) = 1 : 3 : 5 : \dots$$



**Illustration 25.**

A rocket is fired vertically up from the ground with a resultant vertical acceleration of  $10\text{m/s}^2$ . The fuel is finished in 1 minute and it continues to move up.

- What is the maximum height reached?
- After the fuel is finished, calculate the time for which it continues its upwards motion.

(Take  $g = 10\text{ m/s}^2$ )

**Solution**

- The distance travelled by the rocket during burning interval (1 minute = 60s) in which resultant acceleration is vertically upwards and  $10\text{ m/s}^2$  will be  $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000\text{ m} = 18\text{ km}$  and velocity acquired by it will be  $v = 0 + 10 \times 60 = 600\text{ m/s}$

Now after 1 minute the rocket moves vertically up with initial velocity of  $600\text{ m/s}$  and acceleration due to gravity opposes its motion. So, it will go to a height  $h_2$  from this point, till its velocity becomes zero such that  $0 = (600)^2 - 2gh_2$  or  $h_2 = 18000\text{ m} = 18\text{ km}$  [ $g = 10\text{ m/s}^2$ ]

So the maximum height reached by the rocket from the ground,  $H = h_1 + h_2 = 18 + 18 = 36\text{ km}$

- As after burning of fuel the initial velocity  $600\text{m/s}$  and gravity opposes the motion of rocket, so from 1<sup>st</sup> equation of motion time taken by it till its velocity  $v = 0$   
 $0 = 600 - gt \Rightarrow t = 60\text{ s}$

**Illustration 26.**

A ball is thrown upwards from the top of a tower  $40\text{ m}$  high with a velocity of  $10\text{ m/s}$ , find the time when it strikes the ground ( $g = 10\text{ m/s}^2$ )

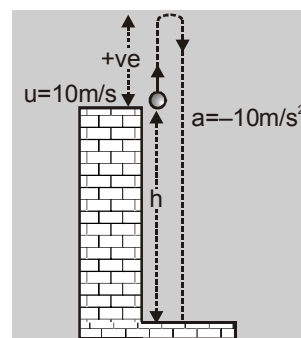
**Solution**

In the problem  $u = +10\text{ m/s}$ ,  $a = -10\text{ m/s}^2$  and  $s = -40\text{ m}$  (at the point where ball strikes the ground)

Substituting in  $s = ut + \frac{1}{2}at^2$

$$-40 = 10t - 5t^2 \text{ or } 5t^2 - 10t - 40 = 0 \quad \text{or} \quad t^2 - 2t - 8 = 0$$

Solving this we have  $t = 4\text{ s}$  and  $-2\text{ s}$ . Taking the positive value  $t = 4\text{ s}$ .

**Illustration 27.**

A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground: (a) earlier (b) with greater speed?

**Solution**

In case of sliding motion on the inclined plane.

$$\frac{h}{s} = \sin \theta \Rightarrow s = \frac{h}{\sin \theta}, \quad a = g \sin \theta$$

$$t_s = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2}{g \sin \theta} \times \frac{h}{\sin \theta}} = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}} = \frac{t_F}{\sin \theta}$$

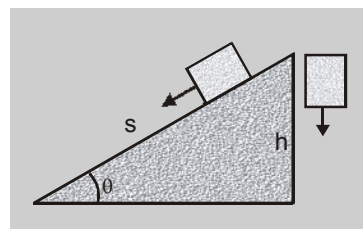
$$v_s = \sqrt{2as} = \sqrt{2g \sin \theta \times \frac{h}{\sin \theta}} = \sqrt{2gh}$$

In case of free fall  $t_F = \sqrt{\frac{2h}{g}}$  and  $v_F = \sqrt{2gh} = v_s$

- $\therefore \sin \theta < 1$ ,  $t_F < t_s$ , i.e., falling body reaches the ground first.

- $v_F = v_s$ , i.e., both reach the ground with same speed.

**Special Note :** (not same velocity, as for falling body direction is vertical while for sliding body along the plane downwards).



**Illustration 28.**

A juggler throws balls into air. He throws one ball whenever the previous one is at its highest point. How high do the balls rise if he throws  $n$  balls each second? Acceleration due to gravity is  $g$ .

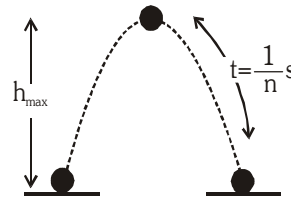
**Solution**

Juggler throws  $n$  balls in one second so time interval between two consecutive throws is  $t = \frac{1}{n}$  s

each ball takes  $\frac{1}{n}$  s to reach maximum height

$$\text{So } h_{\max} = \frac{1}{2} \times g t^2 = \frac{1}{2} \times g \left( \frac{1}{n} \right)^2$$

$$h_{\max} = \frac{g}{2n^2}$$

**Illustration 29.**

A pebble is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2 s. If acceleration due to gravity is 9.8 m/s<sup>2</sup> (a) what is the height of the bridge? (b) with what velocity does the pebble strike the water?

**Solution**

Let height of the bridge be  $h$  then

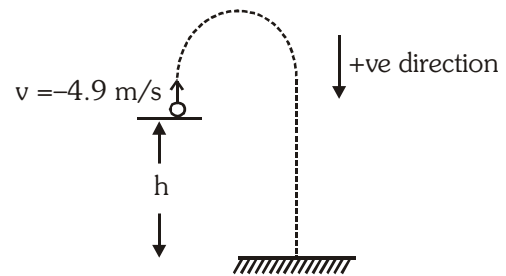
$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2$$

$$\Rightarrow h = 9.8 \text{ m}$$

velocity with which the ball strikes the water surface

$$v = u + at$$

$$\Rightarrow v = -4.9 + 9.8 \times 2 = 14.7 \text{ m/s}$$

**Illustration 30.**

A particle is thrown vertically upwards from the surface of the earth. Let  $T_P$  be the time taken by the particle to travel from a point  $P$  above the earth to its highest point and back to the point  $P$ . Similarly, let  $T_Q$  be the time taken by the particle to travel from another point  $Q$  above the earth to its highest point and back to the same point  $Q$ . If the distance between the points  $P$  and  $Q$  is  $H$ , find the expression for acceleration due to gravity in terms of  $T_P$ ,  $T_Q$  and  $H$ . [AIPMT (Mains) - 2007]

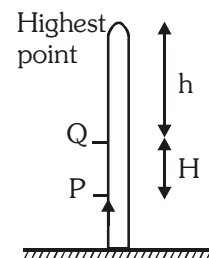
**Solution**

$$\text{Time taken from point P to point P } T_P = 2\sqrt{\frac{2(h+H)}{g}}$$

$$\text{Time taken from point Q to point Q } T_Q = 2\sqrt{\frac{2h}{g}}$$

$$\Rightarrow T_P^2 = \frac{8(h+H)}{g} \text{ \& } T_Q^2 = \frac{8h}{g} \Rightarrow T_P^2 = T_Q^2 + \frac{8H}{g}$$

$$\Rightarrow g = \frac{8H}{T_P^2 - T_Q^2}$$



## BEGINNER'S BOX-6

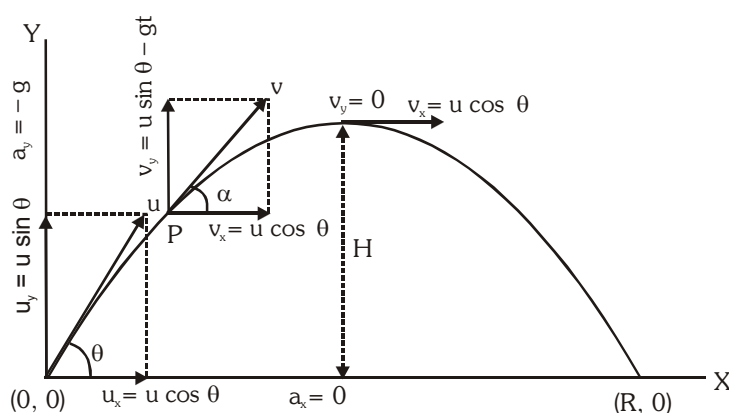
1. A particle is projected vertically upwards from ground with velocity 10 m/s. Find the time taken by it to reach at the highest point ?
2. A particle is projected vertically up from the top of a tower with velocity 10 m/s. It reaches the ground in 5s. Find–
  - (a) Height of tower.
  - (b) Striking velocity of particle at ground
  - (c) Distance traversed by particle.
  - (d) Average speed & average velocity of particle.
3. A balloon starts rising from the ground with an acceleration of  $1.25 \text{ m/s}^2$ . A stone is released from the balloon after 10s. Determine
  - (1) maximum height of stone from ground
  - (2) time taken by stone to reach the ground
4. A rocket is fired vertically up from the ground with an acceleration  $10 \text{ m/s}^2$ . If its fuel is finished after 1 minute then calculate –
  - (a) Maximum velocity attained by rocket in ascending motion.
  - (b) Height attained by rocket before fuel is finished.
  - (c) Time taken by the rocket in the whole motion.
  - (d) Maximum height attained by rocket.
5. A particle is dropped from the top of a tower. During its motion it covers  $\frac{9}{25}$  part of height of tower in the last 1 second Then find the height of tower.
6. A particle is dropped from the top of a tower. It covers 40 m in last 2s. Find the height of the tower.
7. A player throws a ball upwards with an initial speed of 29.4 m/s.
  - (a) What is the direction of acceleration during the upward motion of the ball?
  - (b) What are the velocity and acceleration of the ball at the highest point of its motion?
  - (c) Choose  $x = 0 \text{ m}$  and  $t = 0 \text{ s}$  to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.
  - (d) To what height does the ball rise and how long does the ball take to return to the player's hands ?  
(Take  $g = 9.8 \text{ m/s}^2$  and neglect air resistance).
8. A particle is dropped from the top of a tower. The distance covered by it in the last one second is equal to that covered by it in the first three seconds. Find the height of the tower.
9. Water drops are falling in regular intervals of time from top of a tower of height 9 m. If 4<sup>th</sup> drop begins to fall when 1<sup>st</sup> drop reaches the ground, find the positions of 2<sup>nd</sup> & 3<sup>rd</sup> drops from the top of the tower.



## **PROJECTILE MOTION**

When a body is projected such that velocity of projection is not parallel to the force, then it moves along a curved path. This motion is called two dimensional motion. If force on the body is constant then curved path of the body is parabolic. This motion is studied under projectile motion.

- (i) It is an example of two dimensional motion.
- (ii) It is an example of motion with constant (or uniform) acceleration. Thus equations of motion can be used to analyse projectile motion.
- (iii) A particle thrown in the space which moves under the effect of gravity only is called a "**projectile**". The motion of this particle is referred to as projectile motion.
- (iv) If a particle possesses a uniform acceleration in a directions oblique to its initial velocity, the resultant path will be parabolic. Let X-axis is along the ground and Y-axis is along the vertical then path of projectile projected at an angle  $\theta$  from the ground is as shown.



### **9. GROUND TO GROUND PROJECTION**

Projectile motion can be considered as two mutually perpendicular motions, which are independent of each other. i.e. Projectile motion = Horizontal motion + Vertical motion

#### **Horizontal Motion**

- Initial velocity in horizontal direction =  $u \cos \theta = u_x$
- Acceleration along horizontal direction =  $a_x = 0$ . (Neglect air resistance)
- Therefore, Horizontal velocity remains unchanged.
- At any instant horizontal velocity  $u_x = u \cos \theta$
- At time  $t$ ,  $x$  co-ordinate or displacement along X-direction is

$$x = u_x t \quad \text{or} \quad x = (u \cos \theta) t$$

**Vertical Motion :** It is motion under the effect of gravity so that as particle moves upwards the magnitude of its vertical velocity decreases.

- Initial velocity in vertical direction =  $u \sin \theta = u_y$
- Acceleration along vertical direction =  $a_y = -g$
- At time  $t$ , vertical speed  $v_y = u_y - gt = u \sin \theta - gt$
- In time  $t$ , displacement in vertical direction or "height" of the particle above the ground

$$y = u_y t - \frac{1}{2} g t^2 = u \sin \theta t - \frac{1}{2} g t^2$$



**Net Motion :** Net initial velocity =  $\vec{u} = u_x \hat{i} + u_y \hat{j} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$

Direction of  $u$  can be explained in terms of angle  $\theta$  it makes with the ground

Net acceleration =  $\vec{a} = a_x \hat{i} + a_y \hat{j} = -g \hat{j}$  (direction of  $g$  is downwards)

**Coordinates of particle at time  $t$  :**  $(x, y)$   $x = u_x t$  and  $y = u_y t - \frac{1}{2} g t^2$

Net displacement in  $t$  time =  $\sqrt{x^2 + y^2}$

**Velocity of particle at time  $t$**

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = u_x \hat{i} + (u_y - gt) \hat{j} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

Magnitude of velocity  $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

If angle made by velocity  $\vec{v}$  with the ground is  $\alpha$ , then

$$\tan \alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x}$$

$$\Rightarrow \tan \alpha = \frac{u \sin \theta - gt}{u \cos \theta} = \tan \theta - \frac{gt}{u \cos \theta}$$

### Change in velocity and momentum ( $\vec{p} = m\vec{v}$ ) of projectile

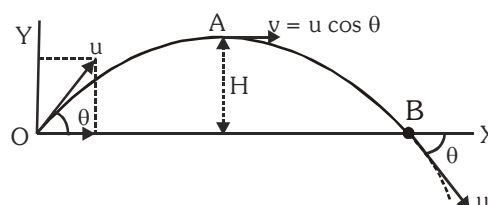
When particle returns to ground again at B point, its  $y$  coordinate is zero and the magnitude of its velocity is  $u$  at angle  $\theta$  with ground. Total angular change =  $2\theta$

Initial velocity  $\vec{u}_i = u \cos \theta \hat{i} + u \sin \theta \hat{j}$

Final velocity  $\vec{u}_f = u \cos \theta \hat{i} - u \sin \theta \hat{j}$

Total change in its velocity,  $|\Delta \vec{v}| = 2u \sin \theta$

Total change in momentum,  $|\Delta \vec{p}| = m|\Delta \vec{v}| = 2mu \sin \theta$



### Time of flight (T)

At time  $T$  particle will be at ground again, i.e. displacement along  $Y$ -axis becomes zero.

$$\therefore y = u_y t - \frac{1}{2} g t^2 \quad \therefore 0 = u_y T - \frac{1}{2} g T^2$$

Time of flight is the time for which projectile remains in air.

$$\text{or } T = \frac{2u_y}{g} = \frac{2u \sin \theta}{g} \quad (\text{neglecting } T = 0)$$

$$\text{Time of ascent} = \text{Time of descent} = \frac{T}{2} = \frac{u_y}{g} = \frac{u \sin \theta}{g}$$

at time  $\frac{T}{2}$  particle attains maximum height of its trajectory.



### Maximum height attained H

At maximum height vertical component of velocity becomes zero. At this instant y coordinate is, its maximum height.

$$\therefore v_y^2 = u_y^2 - 2gy \quad \therefore 0 = u_y^2 - 2gH \quad \{ \because v_y = 0, y = H \}$$

$$H = \frac{u_y^2}{2g} = \frac{u^2 \sin^2 \theta}{2g}$$

### Horizontal range or Range (R)

It is the displacement of particle along X-direction during its complete flight.

$$\therefore x = u_x t \quad \therefore R = u_x T = u_x \frac{2u_y}{g}; \quad R = \frac{2u_x u_y}{g}$$

$$R = \frac{2(u \cos \theta)(u \sin \theta)}{g} \Rightarrow R = \frac{u^2 \sin 2\theta}{g} \quad (\because 2 \sin \theta \cos \theta = \sin 2\theta)$$

### Maximum horizontal range ( $R_{\max}$ )

If value of  $\theta$  is increased from  $\theta = 0^\circ$  to  $90^\circ$ , then range increases from  $\theta = 0^\circ$  to  $45^\circ$  but it decreases beyond  $45^\circ$ . Thus range is maximum at  $\theta = 45^\circ$

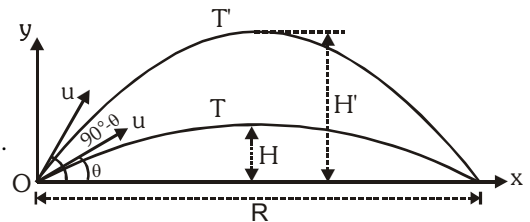
$$\text{For maximum range, } \theta = 45^\circ \text{ and } R_{\max} = \frac{u^2 \sin 2(45^\circ)}{g} = \frac{u^2 \sin 90^\circ}{g}$$

$$\Rightarrow R_{\max} = \frac{u^2}{g}$$

### Comparison of two projectiles of equal range

When two projectiles are thrown with equal speeds at angles  $\theta$  and  $(90^\circ - \theta)$  then their ranges are equal but maximum heights attained are different and time of flights are also different.

$$\text{At angle } \theta, \quad R = \frac{u^2 \sin 2\theta}{g}$$



$$\text{At angle } (90^\circ - \theta), \quad R' = \frac{u^2 \sin 2(90^\circ - \theta)}{g} = \frac{u^2 \sin(180^\circ - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$$

Thus,  $R' = R$

Maximum heights of projectiles

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad \text{and} \quad H' = \frac{u^2 \sin^2 (90^\circ - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$\bullet \quad \frac{H}{H'} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\bullet \quad HH' = \frac{u^4 \sin^2 \theta \cos^2 \theta}{4g^2} = \frac{R^2}{16} \Rightarrow R = 4\sqrt{HH'}$$

$$\bullet \quad H + H' = \frac{u^2 \sin^2 \theta}{2g} + \frac{u^2 \cos^2 \theta}{2g} \Rightarrow H + H' = \frac{u^2}{2g}$$



Time of flight of projectiles

$$T = \frac{2u \sin \theta}{g} ; \quad T' = \frac{2u \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

- $\frac{T}{T'} = \frac{\sin \theta}{\cos \theta} = \tan \theta$

- $TT' = \frac{4u^2 \sin \theta \cos \theta}{g^2} = \frac{2R}{g} \Rightarrow TT' \propto R$

### Equation of Trajectory

Along horizontal direction  $x = u_x t$  or  $x = u \cos \theta t$

Along vertical direction  $y = u_y t - \frac{1}{2} g t^2$  or  $y = u \sin \theta t - \frac{1}{2} g t^2$

On eliminating  $t$  from these two equations

$$y = (u \sin \theta) \left( \frac{x}{u \cos \theta} \right) - \frac{1}{2} g \left( \frac{x}{u \cos \theta} \right)^2$$

$$\Rightarrow y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again  $y = x \tan \theta \left[ 1 - \frac{gx}{2u^2 \sin \theta \cos \theta} \right] = x \tan \theta \left[ 1 - \frac{x}{R} \right]$

## 9.1 Kinetic Energy of a Projectile

$$\text{Kinetic energy} = \frac{1}{2} \times \text{Mass} \times (\text{Speed})^2$$

Let a body is projected with velocity  $u$  at an angle  $\theta$ .

Thus initial kinetic energy of projectile,  $K_0 = \frac{1}{2} m u^2$

Since velocity of projectile at maximum height is  $u \cos \theta$ .

Kinetic energy at highest point,  $K = \frac{1}{2} m (u \cos \theta)^2 = K_0 \cos^2 \theta$

which is the minimum kinetic energy during whole motion.



## GOLDEN KEY POINTS

- At maximum height,  $v_y = 0$  and  $v_x = u_x = u \cos \theta$  so that at maximum height  $v = \sqrt{v_x^2 + v_y^2} = u \cos \theta$
- At maximum height angle between velocity and acceleration is  $90^\circ$ .
- Magnitude of velocity at height 'h'.

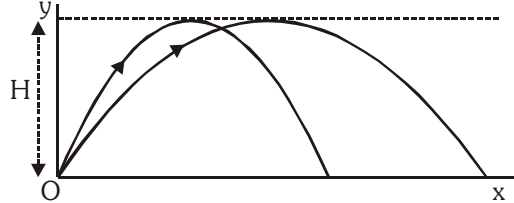
$$v_y^2 = u_y^2 - 2gh$$

$$v_y^2 = (u \sin \theta)^2 - 2gh$$

$$v_x = u \cos \theta$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 \cos^2 \theta + (u \sin \theta)^2 - 2gh}$$

$$|\vec{v}| = \sqrt{u^2 - 2gh}$$



- $T = \frac{2u_y}{g}, \quad H = \frac{u_y^2}{2g}, \quad R = \frac{2u_x u_y}{g}$

T and H depend only upon initial vertical speed  $u_y$

If two projectiles thrown in different directions, have equal times of flight then their initial vertical speeds are same so that their maximum height are is also same.

If  $H_A = H_B$  then  $(u_y)_A = (u_y)_B$  and  $T_A = T_B$

- For situation shown in figure for  $\theta = 45^\circ$

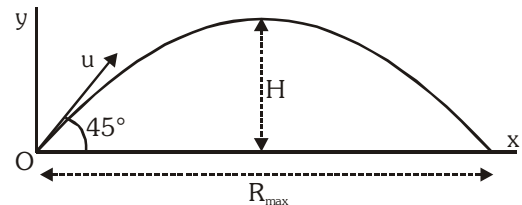
here  $R_{\max} = \frac{u^2}{g}$  and  $H = \frac{u^2 \sin^2 45^\circ}{2g} = \frac{u^2}{4g}$

$$\therefore R_{\max} = 4H = 4 \times (\text{maximum height attained})$$

- When  $R = H$

$$R = \frac{u^2 (2 \sin \theta \cos \theta)}{g} \quad \text{and} \quad H = \frac{u^2 \sin^2 \theta}{2g} \quad \Rightarrow \quad \frac{R}{H} = 4 \cot \theta = 1$$

$$\Rightarrow 4 \cot \theta = 1 \quad \Rightarrow \quad \tan \theta = 4 \quad \Rightarrow \quad \theta = \tan^{-1}(4) \approx 76^\circ$$



## Illustrations

### Illustration 31.

A projectile is thrown with speed  $u$  making angle  $\theta$  with horizontal at  $t=0$ . It just crosses two points of equal height, at time  $t = 1s$  and  $t = 3s$  respectively. Calculate the maximum height attained by it? ( $g=10m/s^2$ )

#### Solution

Displacement in  $y$  direction  $y = u_y \times 1 - \frac{1}{2}g \times (1)^2 = u_y \times 3 - \frac{1}{2}g(3)^2 \Rightarrow u_y = 2g = 20 \text{ m/s}$

Maximum height attained  $h_{\max} = \frac{u_y^2}{2g} = 20m$ .



**Illustration 32.**

A stone is to be thrown so as to cover a horizontal distance of 3m. If the velocity of the projectile is 7 m/s, find :

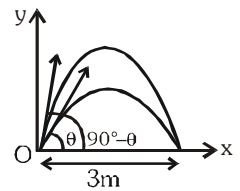
- (a) the angle at which it must be thrown.  
 (b) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

**Solution**

(a) Range  $R = \frac{u^2}{g} \sin 2\theta$

$$\Rightarrow \sin 2\theta = \frac{gR}{u^2} = \frac{9.8 \times 3}{(7)^2} = 0.6 = \sin 37^\circ \Rightarrow 2\theta = 37^\circ \Rightarrow \theta = 18.5^\circ$$

angle of projection may also be  $= 90^\circ - \theta = 90^\circ - 18.5^\circ = 71.5^\circ$



(b) For largest horizontal displacement  $\theta = 45^\circ$  maximum range  $R_{\max} = \frac{u^2}{g} = \frac{(7)^2}{9.8} = \frac{49}{98} \times 10 = 5 \text{ m.}$

**Illustration 33.**

Two projectiles are projected at angles  $(\theta)$  and  $(\frac{\pi}{2} - \theta)$  to the horizontal respectively with same speed 20 m/s. One of them rises 10 m higher than the other. Find the angles of projection. (Take  $g = 10 \text{ m/s}^2$ )

**Solution**

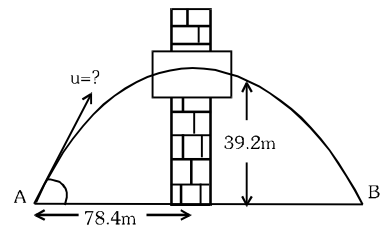
$$\text{Maximum height } H = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow h_1 = \frac{(20)^2 \sin^2 \theta}{2g} = 20 \sin^2 \theta \quad \& \quad h_2 = \frac{(20)^2 \sin^2 (\pi/2 - \theta)}{2g} = 20 \cos^2 \theta$$

$$h_2 - h_1 = 20 [\cos^2 \theta - \sin^2 \theta] = 10 \Rightarrow 20 \cos 2\theta = 10 \Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 60^\circ \Rightarrow \theta = 30^\circ$$

$$\text{and } \theta' = 90^\circ - \theta = 60^\circ$$

**Illustration 34.**

A boy stands 78.4 m away from a building and throws a ball which just enters a window at maximum height 39.2m above the ground. Calculate the velocity of projection of the ball.

**Solution**

$$\text{Maximum height} = \frac{u^2 \sin^2 \theta}{2g} = 39.2 \text{ m} \dots (i) \quad \text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g} = 2 \times 78.4 \dots (ii)$$

from equation (i) divided by equation (ii)  $\tan \theta = 1 \Rightarrow \theta = 45^\circ$

$$\text{from equation (ii) range} = \frac{u^2 \sin 90^\circ}{g} = 2 \times 78.4 \Rightarrow u = \sqrt{2 \times 78.4 \times 9.8} = 39.2 \text{ m/s}$$

**Illustration 35.**

A particle thrown over a triangle from one end of a horizontal base falls on the other end of the base after grazing the vertex. If  $\alpha$  and  $\beta$  are the base angles of triangle and angle of projection is  $\theta$ , then prove that  $\tan \theta = \tan \alpha + \tan \beta$ .

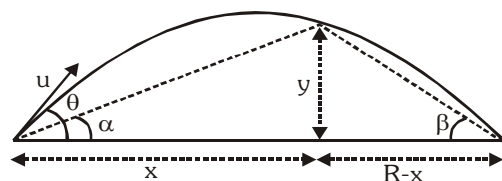
**Solution**

From triangle  $y = x \tan \alpha$  and  $y = (R - x) \tan \beta$

$$\tan \alpha + \tan \beta = \frac{y}{x} + \frac{y}{R - x} = \frac{yR}{x(R - x)}$$

$$\therefore y = x \tan \theta \left[ 1 - \frac{x}{R} \right] \Rightarrow \tan \theta = \frac{yR}{x(R - x)}$$

$$\therefore \tan \theta = \tan \alpha + \tan \beta$$



**Illustration 36.**

A particle is projected from the ground at an angle such that it just clears the top of a pole after  $t_1$  time in its path. It takes further  $t_2$  time to reach the ground. What is the height of the pole ?

**Solution**

Height of the pole is equal to the vertical displacement of the particle at time  $t_1$

$$\text{Vertical displacement } y = u_y t_1 + \frac{1}{2} a_y t_1^2 = u_y t_1 - \frac{1}{2} g t_1^2 \dots\dots\dots(i)$$

$$\text{and total flight time } t_1 + t_2 = \frac{2u_y}{g} \Rightarrow u_y = \frac{g}{2}(t_1 + t_2)$$

$$\text{put value } u_y \text{ in equation (i) } y = \frac{g}{2}(t_1 + t_2)t_1 - \frac{1}{2}g(t_1)^2 = \frac{1}{2}gt_1 t_2, \text{ so height of the pole} = \frac{1}{2}gt_1 t_2$$

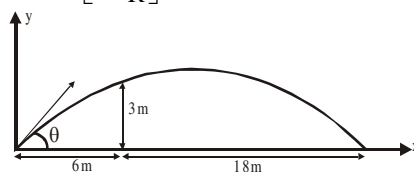
**Illustration 37.**

A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is :-

- (A)  $\tan^{-1} \left( \frac{3}{2} \right)$       (B)  $\tan^{-1} \left( \frac{2}{3} \right)$       (C)  $\tan^{-1} \left( \frac{1}{2} \right)$       (D)  $\tan^{-1} \left( \frac{3}{4} \right)$

**Solution****Ans. (B)**

$$\text{From equation of trajectory, } y = x \tan \theta \left[ 1 - \frac{x}{R} \right] \Rightarrow 3 = 6 \tan \theta \left[ 1 - \frac{1}{4} \right] \Rightarrow \tan \theta = \frac{2}{3}$$

**BEGINNER'S BOX-7**

1. A football player kicks a ball at an angle of  $30^\circ$  to the horizontal with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane, calculate (a) the time at which the ball reaches the highest point (b) the maximum height reached (c) the horizontal range of the ball (d) the time for which the ball is in the air. ( $g = 10 \text{ m/s}^2$ )
2. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the ball, with the same speed ?
3. Two bodies are thrown with the same initial speed at angles  $\alpha$  and  $(90^\circ - \alpha)$  with the horizontal. What will be the ratio of (a) maximum heights attained by them and (b) horizontal ranges ?
4. A ball is thrown at angle  $\theta$  and another ball is thrown at angle  $(90^\circ - \theta)$  with the horizontal direction from the same point each with speeds of 40 m/s. The second ball reaches 50m higher than the first ball. Find their individual heights.  $g = 10 \text{ m/s}^2$ .
5. The range of a particle when launched at an angle of  $15^\circ$  with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle  $45^\circ$  to the horizontal.
6. Show that the projection angle  $\theta_0$  for a projectile launched from the origin is given by :  

$$\theta_0 = \tan^{-1} \left[ \frac{4H_m}{R} \right] \quad \text{Where } H_m = \text{Maximum height, } R = \text{Horizontal range}$$
7. A ball of mass  $m$  is thrown vertically up. Another ball of mass  $2m$  is thrown at an angle  $\theta$  with the vertical. Both of them stay in air for the same periods of time. What is the ratio of the height attained by the two balls ?
8. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the hall ? ( $g = 10 \text{ m/s}^2$ )



## 10. HORIZONTAL PROJECTION FROM HEIGHT

Consider a projectile thrown from point O at some height  $h$  from the ground with a velocity  $u$  in horizontal direction.

Now we shall deal the characteristics of projectile motion separately along horizontal and vertical directions i.e.

Horizontal direction :

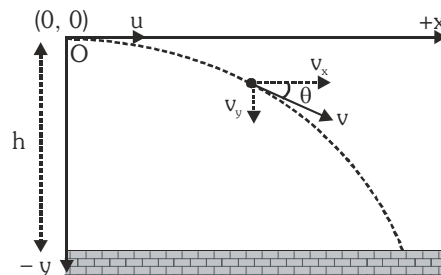
Vertical direction :

(i) Initial velocity  $u_x = u$

Initial velocity  $u_y = 0$

(ii) Acceleration  $a_x = 0$

Acceleration  $a_y = -g$  (downward)



### Trajectory Equation

The path traced by projectile is called its trajectory. After time  $t$ ,

$x = ut$  and  $y = -\frac{1}{2}gt^2$  negative sign indicates that the direction of vertical displacement is downward.

so  $y = -\frac{1}{2}g\frac{x^2}{u^2}$  ( $\because t = \frac{x}{u}$ ) This is equation of a parabola

Above equation is called trajectory equation

### Velocity at a general point P(x, y)

$$v = \sqrt{v_x^2 + v_y^2}$$

Horizontal velocity of the projectile after time  $t$  is  $v_x = u$  (remains constant)

Velocity of projectile in vertical direction after time  $t$  is

$$v_y = 0 - (g)t = -gt \text{ (downward)} \quad \therefore \quad v = \sqrt{u^2 + g^2t^2}$$

and  $\tan \theta = \frac{v_y}{v_x}$  or  $\tan \theta = -\frac{gt}{u}$  (negative sign indicates clockwise direction)

### Displacement

The displacement of the particle is expressed by

$$\vec{s} = x\hat{i} + y\hat{j} \quad \text{Where } |\vec{s}| = \sqrt{x^2 + y^2} = \left| (ut)\hat{i} - \left(\frac{1}{2}gt^2\right)\hat{j} \right|$$

### Time of flight

From equation of motion for vertical direction.

$$h = u_y t + \frac{1}{2}gt^2$$

At highest point  $u_y = 0 \Rightarrow h = \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$

### Horizontal range

Distance covered by the projectile along the horizontal direction between the point of projection to the point

on the ground.  $R = u_x t = u\sqrt{\frac{2h}{g}}$

### Velocity after falling a height $h_1$ :

Along vertical direction  $v_y^2 = 0^2 + 2(h_1)(g)$

$$v_y = \sqrt{2gh_1}$$

Along horizontal direction  $v_x = u_x = u$

$$\text{So velocity, } v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$$



## 11. OBLIQUE PROJECTION FROM A CERTAIN HEIGHT

### (i) Projection from a height at an angle $\theta$ above horizontal :

$$u_x = u \cos \theta \quad u_y = -u \sin \theta$$

$$x = (u \cos \theta) t \quad a_y = g$$

$$H = (-u \sin \theta) t + \frac{1}{2} g t^2$$

$$g t^2 - (2u \sin \theta) t - 2H = 0$$

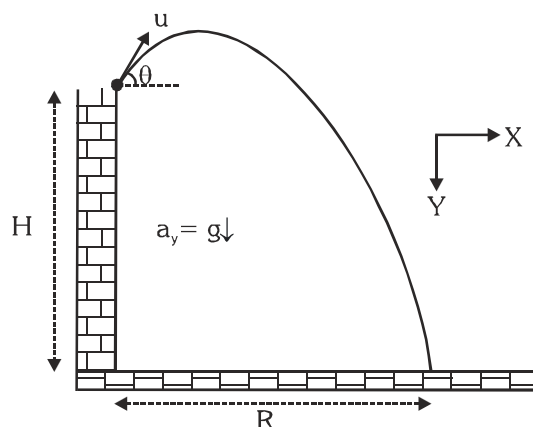
After solving the above equation we get the result

#### Velocity after falling height $h$ :

Along vertical direction;  $v_y^2 = (-u \sin \theta)^2 + 2(h)(g)$

Along horizontal direction,  $v_x = u_x = u \cos \theta$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$



### (ii) Projection from a height at an angle $\theta$ below horizontal :

$$u_x = u \cos \theta \quad u_y = u \sin \theta$$

$$x = (u \cos \theta) t \quad a_y = g$$

$$H = (u \sin \theta) t + \frac{1}{2} g t^2$$

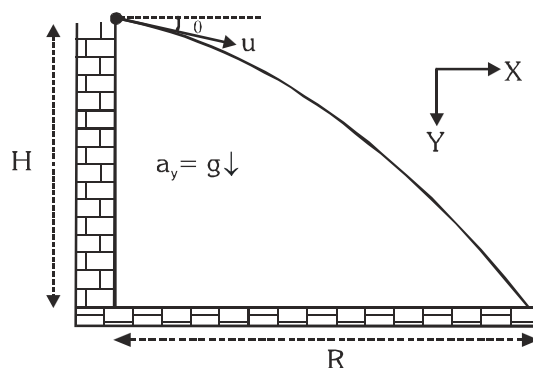
$$g t^2 + (2u \sin \theta) t - 2H = 0$$

After solving the above equation we get the result.

#### Velocity after falling height $h$ :

Along vertical direction,  $v_y^2 = (u \sin \theta)^2 + 2(h)(g)$

Along horizontal direction,  $v_x = u_x = u \cos \theta$ ; So velocity,  $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$



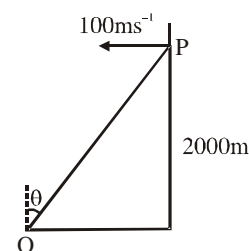
## Illustrations

### Illustration 38.

An aeroplane is travelling horizontally at a height of 2000 m from the ground.

The aeroplane, when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle  $\theta$  must the line PQ make with the vertical ? [ $g = 10 \text{ m/s}^2$ ]

[AIPMT (Mains) 2007]



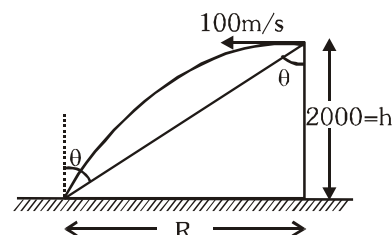
### Solution

Let  $t$  be the time taken by bomb to hit the target.

$$h = 2000 = \frac{1}{2} g t^2 \Rightarrow t = 20 \text{ s}$$

$$R = ut = (100)(20) = 2000 \text{ m}$$

$$\therefore \tan \theta = \frac{R}{h} = \frac{2000}{2000} = 1 \Rightarrow \theta = 45^\circ$$



### Illustration 39.

A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? ( $g = 9.8 \text{ m/s}^2$ )

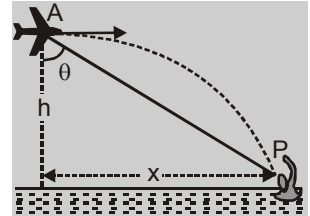
### Solution

Plane is flying at a speed  $= 600 \times \frac{5}{18} = \frac{500}{3} \text{ m/s}$  horizontally (at a height 1960m)

time taken by the kit to reach the ground  $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ s}$

in this time the kit will move horizontally by  $x = ut = \frac{500}{3} \times 20 = \frac{10,000}{3} \text{ m}$

So  $\tan \theta = \frac{x}{h} = \frac{10,000}{3 \times 1960} = \frac{10}{5.88} = 1.7 \approx \sqrt{3}$  or  $\theta = 60^\circ$



### Illustration 40.

A ball rolls off the top of a stair way with a horizontal velocity  $u$ . If each step has height  $h$  and width  $b$  the ball will just hit the edge of  $n^{\text{th}}$  step. Find the value of  $n$ .

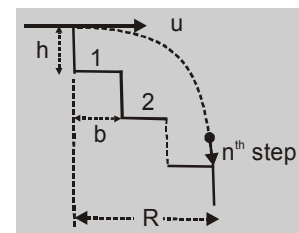
### Solution

If the ball hits the  $n^{\text{th}}$  step, the horizontal and vertical distances traversed are  $nb$  and  $nh$  respectively. Let  $t$  be the time taken by the ball for these horizontal and vertical displacements. Velocity along horizontal direction  $= u$  (remains constant) and initial vertical velocity  $= \text{zero}$ .

$$\therefore \quad nb = ut \quad \text{and} \quad nh = 0 + \frac{1}{2}gt^2$$

Eliminating  $t$  from the equation

$$nh = \frac{1}{2}g \left( \frac{nb}{u} \right)^2 \quad \Rightarrow \quad n = \frac{2hu^2}{gb^2}$$



### Illustration 41.

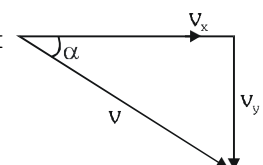
A particle is projected horizontally with a speed 20 m/s from the top of a tower. After what time will the velocity of particle be at  $45^\circ$  angle from the initial direction of projection ? [Let  $g = 10 \text{ m/s}^2$ ]

### Solution

Let  $x$  and  $y$  axes be adopted along horizontal and vertically downward directions respectively.

After time  $t$ ,  $v_x = u_x = 20 \text{ m/s}$ , velocity in  $y$  direction  $v_y = u_y + a_y t = 0 + g t = gt$

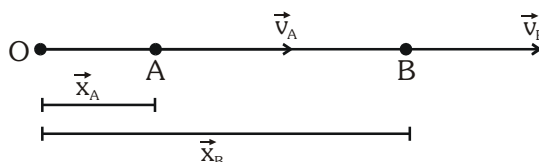
$$\tan \alpha = \frac{v_y}{v_x} = \frac{gt}{20} \quad \text{if } \alpha = 45^\circ \text{ then } 1 = \frac{10 \times t}{20} \Rightarrow t = 2 \text{ s}$$



## BEGINNER'S BOX-8

1. A projectile is fired horizontally with a velocity of  $98 \text{ ms}^{-1}$  from the top of a hill 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the angle at which the projectile hits the ground. ( $g = 9.8 \text{ m/s}^2$ )
2. Two tall buildings face each other and are at a distance of 180m from each other. With what velocity must a ball be thrown horizontally from a window 55m above the ground in one building, so that it enters a window 10m above the ground in the second building ? ( $g = 10 \text{ m/s}^2$ )
3. Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. ( $g = 9.8 \text{ m/s}^2$ )
4. A ball is thrown up from the top of a tower with an initial velocity of  $10 \text{ m/s}$  at an angle of  $30^\circ$  with the horizontal. It hits the ground at a distance of 17.3 m from the base of tower. Calculate the height of the tower. ( $g = 10 \text{ m/s}^2$ )

## 12. RELATIVE VELOCITY IN ONE DIMENSION



Displacement of B with respect to A = Displacement of B as measured from A

$$\Rightarrow \vec{x}_{BA} = \vec{x}_B - \vec{x}_A$$

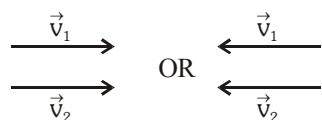
$$\Rightarrow \frac{d\vec{x}_{BA}}{dt} = \frac{d\vec{x}_B}{dt} - \frac{d\vec{x}_A}{dt}$$

$$\Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$\text{Relative} = \text{Actual} - \text{Reference}$$

### For same direction

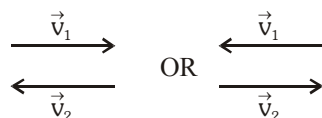
When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$$|\vec{v}_{12}| \text{ or } |\vec{v}_{21}| = v_1 - v_2$$

### For opposite directions

When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal sum of their individual speeds.



$$|\vec{v}_{12}| \text{ or } |\vec{v}_{21}| = v_1 + v_2$$

**Note :-** When two particles move simultaneously then the concept of relative motion becomes applicable conveniently.



## NUMERICAL APPLICATIONS

- (i) When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and in this condition relative velocity is the ratio of relative displacement to time.

$$v_1 = \text{const. } v_2 = \text{const.}$$



$$\text{when } a_{\text{rel.}} = 0$$

$$v_{\text{rel.}} = \frac{s_{\text{Relative}}}{\text{time}}$$

- (ii) When two particles move in such a way that their relative acceleration non zero but constant then we apply equations of motion in the relative form.



$$\begin{aligned} a_{AB} &= a_A - a_B \\ &= 0 - a = -a \neq 0 = \text{constant} \end{aligned}$$

### Equations of Motion (Relative)

- $v_{\text{rel.}} = u_{\text{rel.}} + a_{\text{rel.}} t$
- $s_{\text{rel.}} = u_{\text{rel.}} t + \frac{1}{2} a_{\text{rel.}} t^2$
- $v_{\text{rel.}}^2 = u_{\text{rel.}}^2 + 2 a_{\text{rel.}} s_{\text{rel.}}$
- $s_{\text{rel.}} = \frac{1}{2} (u_{\text{rel.}} + v_{\text{rel.}}) t$

## Illustrations

### Illustration 42.

Buses A and B are moving in the same direction with speeds 20 m/s and 15 m/s respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t. A.

### Solution

Let their direction of motion be along + x-axis then  $\vec{v}_A = (20\text{m/s})\hat{i}$  and  $\vec{v}_B = (15\text{m/s})\hat{i}$

- (a) Relative velocity of A w.r.t. B is  $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (\text{actual velocity of A}) - (\text{velocity of B})$

$$= (20 \text{ m/s})\hat{i} - (15 \text{ m/s})\hat{i} = 5\text{m/s}\hat{i}$$

i.e. A is moving with speed 5 m/s w.r.t B in the same direction.

- (b) Relative velocity of B w.r.t. A is  $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = (\text{actual velocity of B}) - (\text{velocity of A})$

$$= (15 \text{ m/s})\hat{i} - (20 \text{ m/s})\hat{i} = (-5\text{m/s})\hat{i} = (5 \text{ m/s})(-\hat{i})$$

i.e. B is moving in opposite direction w.r.t. A, at a speed 5 m/s



**Illustration 43.**

A police van moving on a highway with a speed of 30 km/hr fires a bullet at a thief's car which is speeding away in the same direction with a speed of 190 km/hr. If the muzzle speed of the bullet is 150 m/s, find the speed of the bullet with respect to the thief's car.

**Solution**  $v_b \rightarrow$  velocity of bullet  
 $v_p \rightarrow$  velocity of police van  
 $v_t \rightarrow$  velocity of thief's car  
 $v_{bp} = v_b - v_p$

$$v_b = v_{bp} + v_p = 150 \times \frac{18}{5} \text{ km/hr} + 30 \text{ km/hr} = 570 \text{ km/hr}$$

$$v_{bt} = v_b - v_t = 570 \text{ km/hr} - 190 \text{ km/hr} = 380 \text{ km/hr}.$$

**Illustration 44.**

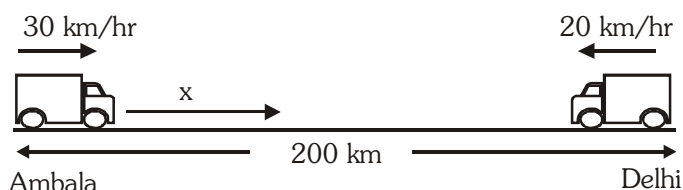
Delhi is at a distance of 200 km from Ambala. Car A sets out from Ambala at a speed of 30 km/hr. and car B set out at the same time from Delhi at a speed of 20 km/hr. When will they cross each other? What is the distance of that crossing point from Ambala?

**Solution**

$$\begin{aligned}\vec{v}_{AB} &= \vec{v}_A - \vec{v}_B \\ &= 30 - (-20) \\ &= 50 \text{ km/hr}\end{aligned}$$

They will meet after time  $t$

$$\text{given by } t = \frac{s}{v_{AB}} = \frac{200}{50} = 4 \text{ hr}$$



Distance from Ambala where they will meet is  $x = 30 \times 4 = 120 \text{ km}$

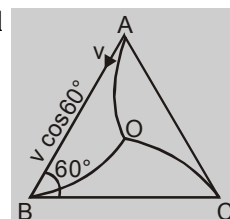
**Illustration 45.**

Three boys A, B and C are situated at the vertices of an equilateral triangle of side  $d$  at  $t = 0$ . Each of the boys move with constant speed  $v$ . A always moves towards B, B towards C and C towards A. When and where will they meet each other?

**Solution**

By symmetry they will meet at the centroid of the triangle. Approaching velocity of A and B towards each other is  $v + v \cos 60^\circ$  and they cover distance  $d$  when they meet. So that time taken, is given by

$$\therefore t = \frac{d}{v + v \cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

**Illustration 46.**

Two cars approach each other on a straight road with velocities 10 m/s and 12 m/s respectively. When they are 150 metres apart, both drivers apply their brakes and each car decelerates at  $2 \text{ m/s}^2$  until they stop. How far apart will they be when both come to a halt?

**Solution**

Let  $x_1$  and  $x_2$  be the distances travelled by the cars before they stop under deceleration.

$$\text{From III}^{\text{rd}} \text{ equation of motion } v^2 = u^2 + 2as, \Rightarrow 0 = (10)^2 - 2 \times 2 \times x_1 \Rightarrow x_1 = 25 \text{ m}$$

$$\text{and } 0 = (12)^2 - 2 \times 2 \times x_2 \Rightarrow x_2 = 36 \text{ m}$$

$$\text{Total distance covered by the two cars} = x_1 + x_2 = 25 + 36 = 61 \text{ m}$$

$$\text{Distance between the two cars when they stop} = 150 - 61 = 89 \text{ m}.$$



### Illustration 47.

Two trains A and B which are 100 m and 60 m long are moving in opposite directions on parallel tracks. Velocity of the shorter train is 3 times that of the longer one. If the trains take 4 seconds to cross each other find the velocities of the trains ?

### Solution

Given that  $v_B = 3v_A$

Trains move in opposite directions then

$$\text{relative velocity } v_{\text{rel}} = \frac{d_{\text{rel}}}{\text{time}} \Rightarrow v_A + v_B = \frac{100 + 60}{4} \Rightarrow 4v_A = 40 \Rightarrow v_A = 10 \text{ m/s}, v_B = 3v_A = 30 \text{ m/s}$$

### BEGINNER'S BOX-9

- Two trains A and B each of length 50 m, are moving with constant speeds. If one train A overtakes the other in 40s, while crosses the other in 20s. Find the speeds of each train.
- Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by  $1 \text{ m/s}^2$ . If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between the guard of B & driver of A ?
- A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?

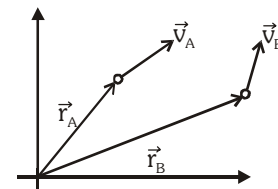
## 13. RELATIVE VELOCITY IN A PLANE

- For 2-dimensional motion :

Relative velocity of A with respect to 'B' can be calculated as

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos \theta}$$



### Note :

- For two particles to collide :
  - their combined relative displacement becomes zero.
  - their combined vertical velocities will be same : if they are projected from same level (in case of projectiles)
  - their combined motion can be converted into two 1D motions.

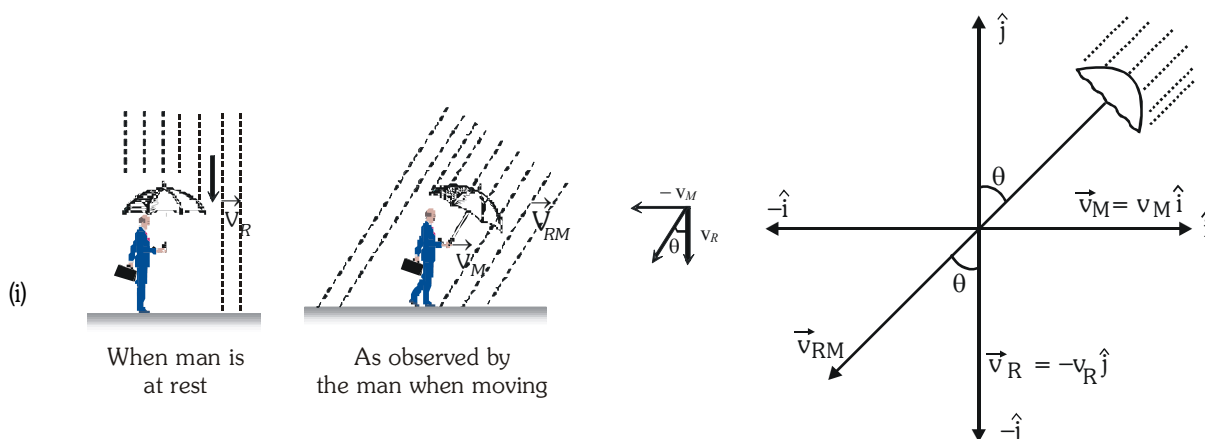
- Relative path of a projectiles w.r.t. another projectile**

Two projectiles are thrown from ground with different velocities at different angles. Since both projectiles have equal accelerations so their relative acceleration is zero. Thus path of one projectile w.r.t. other is a straight line and motion of one projectile w.r.t. other is uniform.

- If  $u_1 \cos \theta_1 = u_2 \cos \theta_2$  then relative path is a vertical line.
- If  $u_1 \sin \theta_1 = u_2 \sin \theta_2$  then relative path is a horizontal line.



## 14. RAIN - MAN PROBLEM



If rain is falling vertically with a velocity  $\vec{v}_R$  and an observer is moving horizontally with speed  $\vec{v}_M$  the velocity of rain relative to observer will be

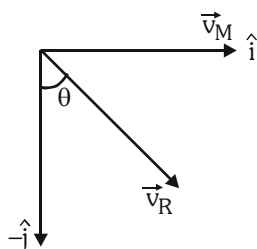
$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = -v_R \hat{j} - v_M \hat{i}$$

which by law of vector addition has magnitude

$$v_{RM} = \sqrt{v_R^2 + v_M^2}$$

The direction of  $\vec{v}_{RM}$  is such that it makes an angle  $\theta$  with the vertical given by  $\theta = \tan^{-1}(v_M/v_R)$  as shown in figure.

- (ii) If rain is already falling at an angle  $\theta$  with the vertical with a velocity  $\vec{v}_R$  and an observer is moving horizontally with speed  $\vec{v}_M$  finds that the rain drops are hitting on his head vertically downwards



Here  $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$$\vec{v}_{RM} = (v_R \sin \theta - v_M) \hat{i} - v_R \cos \theta \hat{j}$$

Now for rain to appear falling vertically, the horizontal component of  $\vec{v}_{RM}$  should be zero, i.e.

$$v_R \sin \theta - v_M = 0 \Rightarrow \sin \theta = \frac{v_M}{v_R}$$

$$\text{and } |\vec{v}_{RM}| = v_R \cos \theta = v_R \sqrt{1 - \sin^2 \theta} = v_R \sqrt{1 - \frac{v_M^2}{v_R^2}} = v_{RM} = \sqrt{v_R^2 - v_M^2}$$



## Illustrations

### Illustration 48.

A person moves due east at a speed 6m/s and feels the wind is blowing towards south at a speed 6m/s.

(a) Find actual velocity of wind blow.

(b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

#### Solution

$$(a) \quad \vec{v}_{act} = \vec{v}_{rel} + \vec{v}_{ref}$$

$$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6\hat{j} + 6\hat{i}$$

$$\vec{v}_w = 6\hat{i} - 6\hat{j}$$

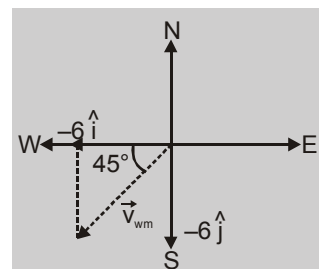
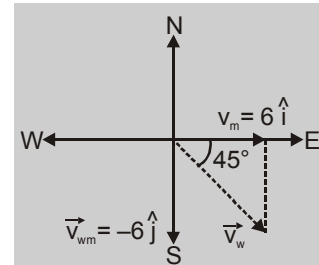
$|\vec{v}| = 6\sqrt{2}$  m/s and it is blowing along S - E

(b) Person doubles its velocity then  $\vec{v}_m = 12\hat{i}$   
but actual wind velocity remains unchanged.

$$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i}$$

$$\vec{v}_{wm} = -6\hat{i} - 6\hat{j}$$

Now relative velocity of wind is  $6\sqrt{2}$ m/s along S - W.



### Illustration 49.

A man at rest observes the rain falling vertically. When he walks at 4 km/h, he has to hold his umbrella at an angle of  $53^\circ$  from the vertical. Find the velocity of raindrops.

#### Solution

Assigning usual symbols  $\vec{v}_m$ ,  $\vec{v}_r$  and  $\vec{v}_{r/m}$  to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation

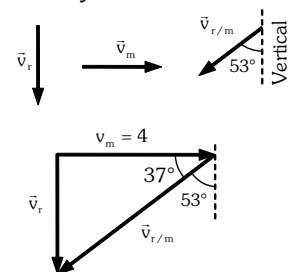
$$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$$

The above equation suggests that a standstill man observes velocity  $\vec{v}_r$  of rain

relative to the ground and while he is moving with velocity  $\vec{v}_m$ , he observes

velocity of rain relative to himself  $\vec{v}_{r/m}$ . It is a common intuitive fact that umbrella must be held against  $\vec{v}_{r/m}$  for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

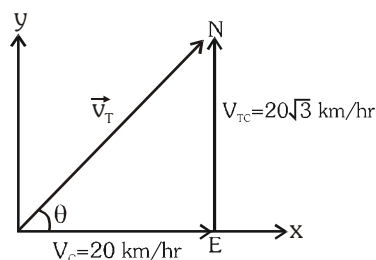
Therefore  $v_r = v_m \tan 37^\circ = 3$  km/h



### Illustration 50.

A man is going east in a car with a velocity of 20 km/hr, a train appears to move towards north to him with a velocity of  $20\sqrt{3}$  km/hr. What is the actual velocity and direction of motion of train?

#### Solution.



$$\vec{v}_{TC} = \vec{v}_T - \vec{v}_C$$

$$\vec{v}_T = \vec{v}_{TC} + \vec{v}_C = 20\sqrt{3}\hat{j} + 20\hat{i}$$

$$|\vec{v}_T| = \sqrt{(20\sqrt{3})^2 + (20)^2} = \sqrt{1600} = 40 \text{ km/hr}$$

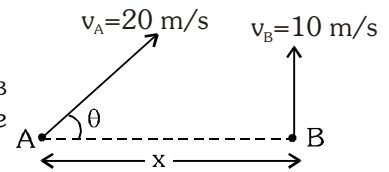
$$\tan \theta = \frac{20\sqrt{3}}{20} \Rightarrow \theta = 60^\circ$$

So direction of motion of train is  $60^\circ$  N of E or  $E-60^\circ$ -N



**Illustration 51.**

Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities  $v_A = 20 \text{ m/s}$  and  $v_B = 10 \text{ m/s}$  respectively. They collide after 0.5 s. Find out the angle  $\theta$  and the distance  $x$ .

**Solution.**

Both particle will collide if they are at same height in same time.

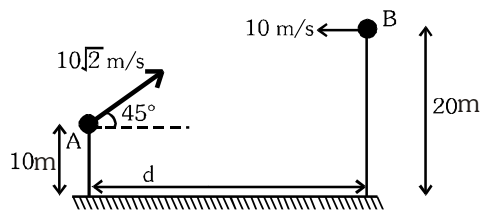
$$y_A = y_B \Rightarrow (u_y)_A t - \frac{1}{2} g t^2 = (u_y)_B t - \frac{1}{2} g t^2 \Rightarrow (u_y)_A = (u_y)_B$$

$$\Rightarrow (v_A \sin \theta) = v_B \Rightarrow 20 \sin \theta = 10 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

In 0.5s horizontal distance covered by A is  $x = (u_x)_A t = (20 \cos 30^\circ) 0.5 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ m}$

**Illustration 52.**

Two particles are projected from the two towers simultaneously, as shown in the figure.



What should be the value of 'd' for their collision ?

**Solution.**

There is no relative acceleration of between A and B.

so time of collision  $t = \frac{y_{BA}}{(v_y)_{BA}}$  where  $y_{BA}$  = vertical displacement of B w.r.t. A = 10 m.

$$(v_y)_{BA} = \text{vertical velocity of B w.r.t. A} = 0 - (-10\sqrt{2} \sin 45^\circ) = 10 \text{ m/s} \Rightarrow t = \frac{10}{10} = 1 \text{ s}$$

$$d = \text{horizontal distance travelled by B w.r.t. A} = (v_x)_{BA} \times t = (10 + 10\sqrt{2} \cos 45^\circ) \times 1 = 20 \text{ m.}$$

**Illustration 53.**

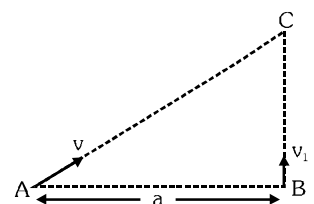
Two boys are standing at the ends A and B of a ground where  $AB = a$ . The boy at B starts running in a direction perpendicular to AB, with velocity  $v_1$ . The boy at A starts running simultaneously with velocity  $v$  and catches the other boy in a time  $t$ , then find  $t$ .

**Solution**

Let the two boys meet at point C after time 't'

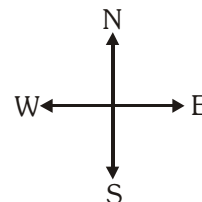
$$\text{Then } AC = vt, BC = v_1 t \text{ but } (AC)^2 = (AB)^2 + (BC)^2 \Rightarrow v^2 t^2 = a^2 + v_1^2 t^2$$

$$\Rightarrow t^2(v^2 - v_1^2) = a^2 \Rightarrow t = \sqrt{\frac{a^2}{v^2 - v_1^2}}$$



## BEGINNER'S BOX-10

1. A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E-30°-N with 10 m/s. Find the relative velocity of B w.r.t. A.
2. A and B are moving with the same speed 10 m/s in the directions E-30°-N and E-30°-S respectively. Find the relative velocity of A w.r.t. B.
3. Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.
  - (a) relative velocity of A w.r.t. B.
  - (b) minimum separation attained during motion
  - (c) time elapsed from starting, to attain minimum separation.
4. Rain is falling vertically with a speed of 30 m/s. A woman rides a bicycle with a speed of 10 m/s in the north to south direction. What is the direction in which she should hold her umbrella?
5. A man is running up hill with a velocity  $(2\hat{i} + 3\hat{j})$  m/s w.r.t. ground. He feels that the rain drops are falling vertically with velocity 4 m/s. If he runs down hill with same speed, find  $v_m$ .
6. A body is projected with velocity  $u_1$  from point A as shown in figure. At the same time another body is projected vertically upwards with a velocity  $u_2$  from point B. What should be the value of  $\frac{u_1}{u_2}$  for both bodies to collide.



## 15. RIVER-BOAT (OR MAN) PROBLEM

A man can swim with velocity  $\vec{v}$ , i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with velocity  $\vec{v}_R$  then velocity of man relative to ground  $\vec{v}_m = \vec{v} + \vec{v}_R$

- (i) If the swimming is in the direction of flow of water or along the downstream then

$$v_m = v + v_R$$



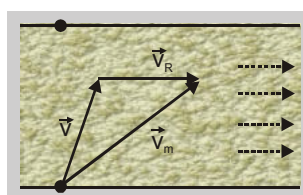
- (ii) If the swimming is in the direction opposite to the flow of water or along the upstream then

$$v_m = v - v_R$$



- (iii) If the man is crossing the river i.e.  $\vec{v}$  and  $\vec{v}_R$  are non collinear then use vector algebra.

$$\vec{v}_m = \vec{v} + \vec{v}_R$$



## To Cross a River

Minimum distance of approach

OR

Cross the river along shortest possible path

OR

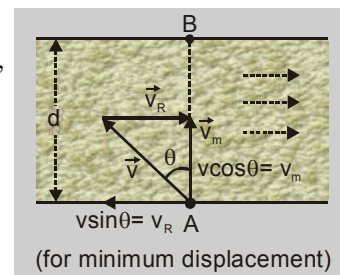
Cross the river and reach a point just opposite to the starting path.

Cross the river in shortest possible time

### For shortest path :

If a man wants to cross the river such that his "displacement should be minimum", it means he intends to reach just opposite point across the river. He should start swimming at an angle  $\theta$  with the perpendicular to the flow of river towards upstream.

such that its resultant velocity  $\vec{v}_m = (\vec{v} + \vec{v}_R)$  It is in the direction of displacement AB.



To reach at B  $v \sin \theta = v_R \Rightarrow \sin \theta = \frac{v_R}{v}$

component of velocity of man along AB is  $v \cos \theta$

so time taken

$$T = \frac{d}{v \cos \theta} = \frac{d}{\sqrt{v^2 - v_R^2}}$$

### For minimum time

To cross the river in minimum time, the velocity along

AB ( $v \cos \theta$ ) should be maximum.

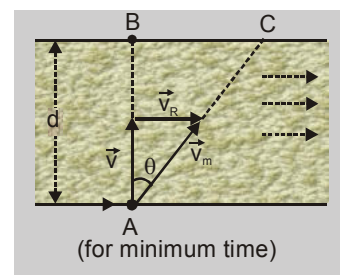
It is possible if  $\theta = 0$ , i.e. swimming should start perpendicular to water current.

Due to effect of river velocity man will reach at point C along resultant velocity, i.e. his displacement will not be minimum but time taken to cross the river will be minimum,

$$t_{\min} = \frac{d}{v}$$

In time  $t_{\min}$  swimmer travels distance BC along the river with speed of river  $v_R \therefore BC = t_{\min} v_R$

distance travelled along river flow = drift of man =  $t_{\min} v_R = \frac{d}{v} v_R$



## Illustrations

### Illustration 54.

A boat moves along the flow of river between two fixed points A and B. It takes  $t_1$  time when going downstream and takes  $t_2$  time when going upstream between these two points. What time it will take in still water to cover the distance equal to AB.

### Solution

$$t_1 = \frac{AB}{v_b + v_R}, t_2 = \frac{AB}{v_b - v_R} \quad \text{or} \quad v_b + v_R = \frac{AB}{t_1} \quad \text{and} \quad v_b - v_R = \frac{AB}{t_2}$$

$$\Rightarrow 2v_b = \frac{AB}{t_1} + \frac{AB}{t_2} = AB \left( \frac{t_1 + t_2}{t_1 t_2} \right)$$

$$\text{or} \quad \left( \frac{2t_1 t_2}{t_1 + t_2} \right) = \frac{AB}{v_b} = \text{time taken by the boat to cover AB}$$



**Illustration 55.**

A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

- In which direction must it be steered to cross the river perpendicular to current?
- How long will it take to cross the river in a direction perpendicular to the river flow?
- In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

**Solution**

- To cross the river perpendicular to current i.e. along shortest path

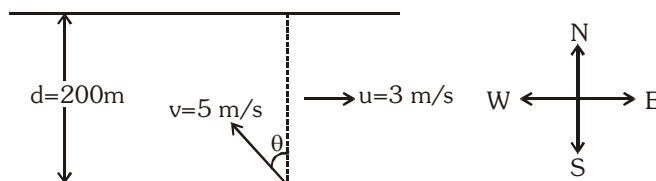
$$\sin \theta = \frac{u}{v} = \frac{3}{5} \Rightarrow \theta = 37^\circ$$

$$(b) \text{ Time taken by boat, } t = \frac{d}{v \cos \theta} = \frac{200}{5 \times \frac{4}{5}} = 50s$$

- To cross the river in minimum time,  $\theta = 0^\circ$

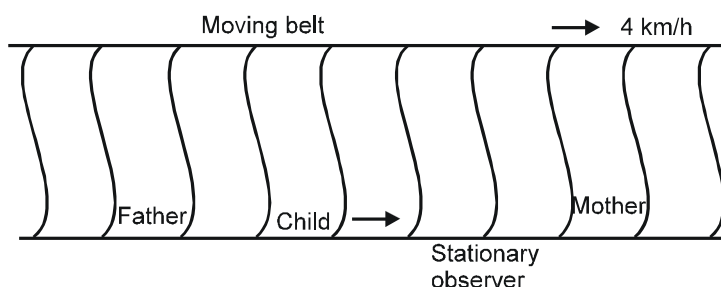
$$\text{Therefore } t_{\min} = \frac{d}{v} = \frac{200}{5} s = 40s$$

$$\text{Drift} = u(t_{\min}) = 3(40)m = 120 m$$

**BEGINNER'S BOX-11**

- A man can swim at a speed 2 m/s in still water. He starts swimming in a river at an angle  $150^\circ$  to the direction of water flow and reaches the directly opposite point on the opposite bank.
  - Find the speed of flowing water.
  - If width of river is 1 km then calculate the time taken to cross the river.
- 2 km wide river flowing at the rate of 5 km/hr. A man can swim in still water with 10 km/hr. He wants to cross the river along the shortest path. Find –
  - in which direction should the person swim.
  - crossing time.
- A child runs to and fro with a speed 9 km/h (with respect to the belt) on a long horizontally moving belt (fig.) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km/h. For an observer on stationary platform outside, what is the
  - speed of the child while running in the direction of motion of the belt?
  - speed of the child while running opposite to the direction of motion of the belt?
  - time taken by the child in (a) and (b)?

Which of the answers will alter if motion is viewed by one of the parents?



## ANSWERS

### BEGINNER'S BOX-1

1.  $7\pi r$ ,  $2r$
2. 400 m for each, B.
3.  $R\sqrt{\pi^2 + 4}$
4. 19m, 13m
5.  $(40\pi)m$ , 80m from A to B
6. (A) 110m (B) 50m,  $37^\circ N$  of E

### BEGINNER'S BOX-2

1. (i) 40 km/hr (ii) 32.5 km/hr (iii) 1040 km/hr (iv) 1040 km/hr
2. 4 m/s
3.  $5 \times 10^{-3} \text{ cm/s}$ ,  $2 \times 10^{-4} \text{ cm/s}$
4. 0 m/s, 20 m/s
5. (a) 12.5 m/s (b) 25 m/s
6. 2 m/s
7. 49.3 km/h, 21.4 km/h

### BEGINNER'S BOX-3

1.  $\frac{10}{\pi} \text{ m/s}^2$
2. (a) 68, (b) 14, (c) 33, (d) 14

### BEGINNER'S BOX-4

1.  $s_2 = 8s_1$
2.  $\sqrt{\frac{u^2 + v^2}{2}}$
3.  $\frac{n^2}{2n-1}$
4.  $3.06 \text{ ms}^{-2}$ ; 11.4 s
5.  $ut + \frac{u^2}{2a}$
6.  $0.218 \text{ m/s}^2$

### BEGINNER'S BOX-5

1. 1 : 3
2. (i) 120m (ii) 0 (iii) 20m/s (iv) 0
3. (a) A, B, (b) A,B, (c) B, A, (d) Same, (e) B, A, once
4. (i) 37 m (ii) 3.7 m/s (iii) Part BC,  $a = 8 \text{ m/s}^2$  (iv)  $2 \text{ m/s}^2$
5. 625 m

### BEGINNER'S BOX-6

1. 1 s
2. (a) 75m, (b) 40m/s, (c) 85m, (d) 17m/s, 15m/s
3. (i) 70.3m (ii) 5 s
4. (a) 600 m/s, (b) 18 km, (c)  $(2 + \sqrt{2})$  min, (d) 36 km
5. 125 m
6. 45 m
7. (a) Vertically downwards; (b) zero velocity, acceleration of  $9.8 \text{ m/s}^2$  downwards.  
(c)  $x > 0$  (upward and downward motion)  $v < 0$  (upward),  $v > 0$  (downward),  $a > 0$  throughout;  
(d) 44.1 m, 6s.
8. 125m
9. 4m & 1m from top

### BEGINNER'S BOX-7

1. (a) 1s, (b) 5m, (c) 34.64 m, (d) 2s
2. 50 m.
3. (a)  $\tan^2 \alpha$  (b) 1
4. 15 m & 65 m
5. 3 km
7. 1
8. 148.32 m

### BEGINNER'S BOX-8

1. (i) 10s, (ii) 980m, (iii)  $\beta = 45^\circ$
2. 60 m/s
3. 700 m/s
4. 10 meter.

### BEGINNER'S BOX-9

1. 3.75 m/s & 1.25 m/s
2. 1250 m.
3. 10 sec, 10 sec

### BEGINNER'S BOX-10

1.  $5\sqrt{3}\hat{i} - 5\hat{j}$ , E-  $30^\circ$  - S
2. In north direction 10m/s
3.  $20\sqrt{2} \text{ km/hr}$  S - E,  $5\sqrt{2} \text{ km}$ , 15 min.
4.  $\alpha = \tan^{-1}\left(\frac{1}{3}\right) = 18.4^\circ$  from vertical towards south
5.  $\sqrt{20} \text{ m/s}$
6.  $\frac{2}{\sqrt{3}}$

### BEGINNER'S BOX-11

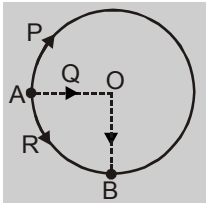
1. (a)  $\sqrt{3} \text{ m/s}$   
(b) 1000s.
2. (a) Direction from Down Stream =  $120^\circ$   
(b)  $\frac{2}{5\sqrt{3}} \text{ hr.}$
3. (a) 13 km/h  
(b) 5 km/h  
(c) 20s

if the motion is viewed by one of the parents answers to (a) and (b) are altered while answer to (c) remain unchanged.



## EXERCISE-I (Conceptual Questions)

### DISTANCE & DISPLACEMENT, SPEED & VELOCITY, AVERAGE SPEED & AVERAGE VELOCITY

1. A man walks 30 m towards north, then 20 m towards east and in the last  $30\sqrt{2}$  m towards south - west. The displacement from origin is :  
(1) 10 m towards west  
(2) 10 m towards east  
(3)  $60\sqrt{2}$  m towards north west  
(4)  $60\sqrt{2}$  m towards east north
2. A body moves along the curved path of a quarter circle. Calculate the ratio of distance to displacement:  
(1) 11 : 7  
(2) 7 : 11  
(3)  $11 : \sqrt{2} \times 7$   
(4)  $7 : 11\sqrt{2}$
3. Three particles P, Q and R are situated at point A on the circular path of radius 10 m. All three particles move along different paths and reach point B as shown in figure. Then the ratio of distance traversed by particles P and Q is :  
  
(1)  $\frac{3}{4}$   
(2)  $\frac{1}{3}$   
(3)  $\frac{3\pi}{4}$   
(4)  $\frac{\pi}{3}$
4. If displacement of a particle is zero, the distance covered :  
(1) must be zero  
(2) may or may not be zero  
(3) cannot be zero  
(4) depends upon the particle
5. If the distance covered is zero, the displacement :  
(1) must be zero  
(2) may or may not be zero  
(3) cannot be zero  
(4) depends upon the particle

6. The location of a particle is changed. What can we say about the displacement and distance covered by the particle :  
(1) Both cannot be zero  
(2) One of the two may be zero  
(3) Both must be zero  
(4) If one is positive, the other is negative and vice-versa
7. An athlete completes one round of a circular track of radius R in 20 seconds. What will be his displacement at the end of 2 minutes 20 seconds ?  
(1) Zero  
(2) 2R  
(3)  $2\pi R$   
(4)  $7\pi R$
8. A man walks for some time 't' with velocity (v) due east. Then he walks for same time 't' with velocity (v) due north. The average velocity of the man is :  
(1) 2v  
(2)  $\sqrt{2} v$   
(3) v  
(4)  $\frac{v}{\sqrt{2}}$
9. A drunkard is walking along a straight road. He takes 5 steps forward and 3 steps backward, followed by 5 steps forward and 3 steps backward and so on. Each step is one meter long and takes one second. There is a pit on the road 11 meters away from the starting point. The drunkard will fall into the pit after :  
(1) 29 s  
(2) 21 s  
(3) 37s  
(4) 31 s
10. A car runs at constant speed on a circular track of radius 10 m taking 6.28s on each lap (i.e. round). The average speed and average velocity for each complete lap is :  
(1) Velocity 10 m/s, speed 10 m/s  
(2) Velocity zero, speed 10 m/s  
(3) Velocity zero, speed zero  
(4) Velocity 10 m/s speed zero
11. A particle moving in a straight line covers half the distance with speed of 12 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is :  
(1) 8.0 m/s  
(2) 12.0 m/s  
(3) 10.0 m/s  
(4) 9.8 m/s



- 12.** The magnitude of average velocity is equal to the average speed when a particle moves :  
 (1) on a curved path  
 (2) in the same direction  
 (3) with constant acceleration  
 (4) with constant retardation
- 13.** A body covers one-third of the distance with a velocity  $v_1$  the second one-third of the distance with a velocity  $v_2$ , and the last one-third of the distance with a velocity  $v_3$ . The average velocity is :  
 (1)  $\frac{v_1 + v_2 + v_3}{3}$   
 (2)  $\frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$   
 (3)  $\frac{v_1v_2 + v_2v_3 + v_3v_1}{3}$   
 (4)  $\frac{v_1v_2v_3}{3}$
- 14.** A car travels a distance  $d$  on a straight road in two hours and then returns to the starting point in next three hours. Its average speed is :  
 (1)  $\frac{d}{5}$  (2)  $\frac{2d}{5}$   
 (3)  $\frac{d}{2} + \frac{d}{3}$  (4) none of these
- 15.** A train covers the first half of the distance between two stations with a speed of 40 km/h and the other half with 60 km/h. Then its average speed is :  
 (1) 50 km/h (2) 48 km/h  
 (3) 52 km/h (4) 100 km/h
- 16.** A car moving on a straight road covers one third of a certain distance with 20 km/h and the rest with 60 km/h. The average speed is :  
 (1) 40 km/h  
 (2) 80 km/h  
 (3)  $46\frac{2}{3}$  km/h  
 (4) 36 km/h
- 17.** A particle moves in the east direction with 15 m/sec for 2 sec then northwards with 5 m/s for 8 sec. Average speed of the particle is :—  
 (1) 1 m/s (2) 5 m/s  
 (3) 7 m/s (4) 10 m/s
- 18.** The numerical ratio of displacement to the distance covered is always :—  
 (1) less than one  
 (2) equal to one  
 (3) equal to or less than one  
 (4) equal to or greater than one
- 19.** A particle moves in a straight line for 20 seconds with velocity 3 m/s and then moves with velocity 4 m/s for another 20 seconds and finally moves with velocity 5 m/s for next 20 seconds. What is the average velocity of the particle ?  
 (1) 3 m/s (2) 4 m/s (3) 5 m/s (4) zero
- 20.** An object travels 10 km at a speed of 100 m/s and another 10 km at 50 m/s. The average speed over the whole distance is :—  
 (1) 75 m/s (2) 55 m/s  
 (3) 66.7 m/s (4) 33.3 m/s
- 21.** A point object traverses half the distance with velocity  $v_0$ . The remaining part of the distance is covered with velocity  $v_1$  for the half time and with velocity  $v_2$  for the rest half. The average velocity of the object for the whole journey is  
 (1)  $2v_1(v_0 + v_2) / (v_0 + 2v_1 + 2v_2)$   
 (2)  $2v_1(v_0 + v_1) / (v_0 + v_1 + v_2)$   
 (3)  $2v_0(v_1 + v_2) / (v_1 + v_2 + 2v_0)$   
 (4)  $2v_2(v_0 + v_1) / (v_1 + 2v_2 + v_0)$
- 22.** Select the incorrect statements from the following.  
 S1 : Average velocity is path length divided by time interval.  
 S2. In general, average speed is greater than the magnitude of the average velocity  
 S3. A particle moving in a given direction with a non-zero velocity can have zero speed.  
 S4. The magnitude of average velocity is the average speed.  
 (1) S2 and S3  
 (2) S1 and S4  
 (3) S1, S3 and S4  
 (4) All four statements

### ACCELERATION, AVERAGE ACCELERATION & APPLICATION OF CALCULUS

- 23.** If  $x$  denotes displacement in time  $t$  and  $x = a \cos t$ , then acceleration is :  
 (1)  $a \cos t$  (2)  $-a \cos t$   
 (3)  $a \sin t$  (4)  $-a \sin t$



- 24.** The velocity-time relation of an electron starting from rest is given by  $u = kt$ , where  $k = 2 \text{ m/s}^2$ . The distance traversed in 3 sec is :  
 (1) 9m (2) 16 m (3) 27 m (4) 36 m
- 25.** The position  $x$  of a particle varies with time ( $t$ ) as  $x = at^2 - bt^3$ . The acceleration at time  $t$  of the particle will be equal to zero, where  $t$  is equal to :  
 (1)  $\frac{2a}{3b}$  (2)  $\frac{a}{b}$  (3)  $\frac{a}{3b}$  (4) zero
- 26.** A particle moves along a straight line such that its displacement at any time  $t$  is given by  $s = t^3 - 6t^2 + 3t + 4$  metres. The velocity when the acceleration is zero is :  
 (1) 3 m/s (2) -12 m/s (3) 42 m/s (4) -9 m/s
- 27.** The displacement of a particle starting from rest (at  $t=0$ ) is given by  $s = 6t^2 - t^3$ . The time when the particle will attain zero velocity again, is :  
 (1) 4s (2) 8s (3) 12s (4) 16s
- 28.** The velocity of a body depends on time according to the equation  $v = 20 + 0.1t^2$ . The body has :  
 (1) uniform acceleration  
 (2) uniform retardation  
 (3) non-uniform acceleration  
 (4) zero acceleration
- 29.** The displacement of a particle is given by  $y = a + bt + ct^2 - dt^4$ . The initial velocity and acceleration are respectively :  
 (1)  $b, -4d$  (2)  $-b, 2c$   
 (3)  $b, 2c$  (4)  $2c, -4d$
- 30.** The initial velocity of a particle is  $u$  (at  $t = 0$ ) and the acceleration is given by  $f = at$ . Which of the following relations is valid ?  
 (1)  $v = u + at^2$  (2)  $v = u + \frac{at^2}{2}$   
 (3)  $v = u + at$  (4)  $v = u$
- 31.** A particle located at  $x = 0$  at time  $t = 0$ , starts moving along the positive  $x$ -direction with a velocity ' $v$ ' which varies as  $v = \alpha\sqrt{x}$ , then velocity of particle varies with time as : ( $\alpha$  is a constant)  
 (1)  $v \propto t$  (2)  $v \propto t^2$   
 (3)  $v \propto \sqrt{t}$  (4)  $v = \text{constant}$
- 32.** The relation  $t = \sqrt{x} + 3$  describes the position of a particle where  $x$  is in meters and  $t$  is in seconds. The position, when velocity is zero, is :-  
 (1) 2 m (2) 4 m (3) 5 m (4) zero
- 33.** The displacement of a particle is represented by the following equation :  $s = 3t^3 + 7t^2 + 5t + 8$  where  $s$  is in metres and  $t$  in seconds. The acceleration of the particle at  $t = 1$  s is :-  
 (1)  $14 \text{ m/s}^2$  (2)  $18 \text{ m/s}^2$   
 (3)  $32 \text{ m/s}^2$  (4) zero
- 34.** If for a particle position  $x \propto t^2$  then :-  
 (1) velocity is constant  
 (2) acceleration is constant  
 (3) acceleration is variable  
 (4) None of these
- 35.** A body is moving according to the equation  $x = at + bt^2 - ct^3$ . Then its instantaneous speed is given by :-  
 (1)  $a + 2b + 3ct$  (2)  $a + 2bt - 3ct^2$   
 (3)  $2b - 6ct$  (4) None of these
- 36.** The motion of a particle is described by the equation  $x = a + bt^2$  where  $a = 15 \text{ cm}$  and  $b = 3 \text{ cm/sec}^2$ . Its instantaneous velocity at time 3 sec will be :-  
 (1)  $36 \text{ cm/sec}$  (2)  $18 \text{ cm/sec}$   
 (3)  $16 \text{ cm/sec}$  (4)  $32 \text{ cm/sec}$
- 37.** Starting from rest, the acceleration of a particle is  $a = 2(t - 1)$ . The velocity of the particle at  $t = 5 \text{ s}$  is :-  
 (1)  $15 \text{ m/s}$  (2)  $25 \text{ m/s}$   
 (3)  $5 \text{ m/s}$  (4) None of these
- 38.** Which of the following equations represents the motion of a body moving with constant finite acceleration? in these equations,  $y$  denotes the displacement in time  $t$  and  $p, q$  and  $r$  are arbitrary constants :  
 (1)  $y = (p + qt)^2 (r + pt)$   
 (2)  $y = p + tqr$   
 (3)  $y = (p + t)(q + t)(r + 1)$   
 (4)  $y = (p + qt)r$
- 39.** Which of the following relations representing displacement  $x$  of a particle describes motion with constant acceleration ?  
 (1)  $x = 6 - 7t^2$  (2)  $x = 3t^2 + 5t^3 + 7$   
 (3)  $x = 9t^2 + 8$  (4)  $x = 4t^{-2} + 3t^{-1}$



40. Equation of a particle moving along the x axis is  $x = u(t - 2) + a(t - 2)^2$

- (1) the initial velocity of the particle is u
- (2) the acceleration of the particle is a
- (3) the acceleration of the particle is 2a
- (4) at  $t = 2$  particle is not at origin

#### CONSTANT ACCELERATION MOTION, FREEFALL

41. The velocity of a particle moving with constant acceleration at an instant  $t_0$  is 10 m/s. After 5 seconds of that instant the velocity of the particle is 20 m/s. The velocity at 3 second before  $t_0$  is :
- (1) 8 m/s
  - (2) 4 m/s
  - (3) 6 m/s
  - (4) 7 m/s
42. The velocity acquired by a body moving with uniform acceleration is 30 m/s in 2 seconds and 60 m/s in 4 seconds. The initial velocity is :
- (1) zero
  - (2) 2 m/s
  - (3) 4 m/s
  - (4) 10 m/s
43. If a body starts from rest, the time in which it covers a particular displacement with uniform acceleration is :
- (1) inversely proportional to the square root of the displacement
  - (2) inversely proportional to the displacement
  - (3) directly proportional to the displacement
  - (4) directly proportional to the square root of the displacement
44. A body at rest is imparted motion to move in a straight line. It is then obstructed by an opposite force, then:
- (1) the body may necessarily change direction
  - (2) the body is sure to slow down
  - (3) the body will necessarily continue to move in the same direction at the same speed
  - (4) none of the above.
45. If a car at rest accelerates uniformly to a speed of 144 km/h in 20 seconds, it covers a distance of :
- (1) 20 m
  - (2) 400 m
  - (3) 1440 m
  - (4) 2980 m
46. A body starts from rest and with a uniform acceleration of  $10 \text{ ms}^{-2}$  for 5 seconds. During the next 10 seconds it moves with uniform velocity. The total distance travelled by the body is :-
- (1) 100 m
  - (2) 125 m
  - (3) 500 m
  - (4) 625 m

47. Initially a body is at rest. If its acceleration is  $5 \text{ ms}^{-2}$  then the distance travelled in the 18<sup>th</sup> second is :-
- (1) 86.6 m
  - (2) 87.5 m
  - (3) 88 m
  - (4) 89 m

48. If a body starts from rest and travels 120m in the 8<sup>th</sup> second, then acceleration is :
- (1)  $16 \text{ m/s}^2$
  - (2)  $10 \text{ m/s}^2$
  - (3)  $0.227 \text{ m/s}^2$
  - (4)  $0.03 \text{ m/s}^2$

49. If a train travelling at 72 km/h is to be brought to rest in a distance of 200 m, then its retardation should be :
- (1)  $20 \text{ m/s}^2$
  - (2)  $2 \text{ m/s}^2$
  - (3)  $10 \text{ m/s}^2$
  - (4)  $1 \text{ m/s}^2$

50. A car moving with a speed of 40 km/h can be stopped by applying brakes after at least 2m. If the same car is moving with a speed of 80 km/h., what is the minimum stopping distance ?
- (1) 2 m
  - (2) 4 m
  - (3) 6 m
  - (4) 8 m

51. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20m. If the velocity of the car is 30 m/s. It can be stopped by this force in :

- (1)  $\frac{20}{3} \text{ m}$
- (2) 20 m
- (3) 60 m
- (4) 180 m

52. If a car at rest accelerates uniformly and attains a speed of 72 km/h in 10s, then it covers a distance of

- (1) 50 m
- (2) 100 m
- (3) 200 m
- (4) 400 m

53. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by.

- (1)  $T = \frac{2h}{v}$
- (2)  $T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$
- (3)  $T = \sqrt{\frac{2h}{v}} + \frac{h}{g}$
- (4)  $T = \sqrt{\frac{h}{2g}} + \frac{2h}{v}$

54. A stone thrown upwards with a speed 'u' from the top of the tower reaches the ground with a velocity '3u'. The height of the tower is :

- (1)  $\frac{3u^2}{g}$
- (2)  $\frac{4u^2}{g}$
- (3)  $\frac{6u^2}{g}$
- (4)  $\frac{9u^2}{g}$



55. A stone falls from a balloon that is descending at a uniform rate of  $12 \text{ ms}^{-1}$ . The displacement of the stone from the point of release after 10 seconds is :  
( $g = 9.8 \text{ m/s}^2$ )  
(1) 490 m (2) 510 m  
(3) 610 m (4) 725 m
56. A rocket is fired vertically from the ground. It moves upwards with a constant acceleration of  $10 \text{ m/s}^2$ . After 30 seconds the fuel is finished. After what time from the instant of firing the rocket will it attain the maximum height ?  $g = 10 \text{ m/s}^2$  :  
(1) 30 s (2) 45 s  
(3) 60 s (4) 75 s
57. A body is released from the top of a tower of height  $H$  metres. It takes  $t$  time to reach the ground. Where is the body  $\frac{t}{2}$  time after the release :  
(1) At  $\frac{H}{2}$  metres from ground  
(2) At  $\frac{H}{4}$  metres from ground  
(3) At  $\frac{3H}{4}$  metres from the ground  
(4) At  $\frac{H}{6}$  metres from the ground
58. A body dropped from the top of a tower covers a distance  $7x$  in the last second of its journey, where  $x$  is the distance covered in first second. How much time does it take to reach the ground ?  
(1) 3s (2) 4s  
(3) 5s (4) 6s
59. A body falling from height 'h' takes  $t_1$  time to reach the ground. The time taken to cover the first half of the height is :  
(1)  $t_2 = \frac{t_1}{\sqrt{2}}$  (2)  $t_2 = \frac{t_1}{\sqrt{2}}$   
(3)  $t_2 = \sqrt{3} t_1$  (4) None of these
60. Two balls are dropped from different heights at different instants. Second ball is dropped 2 seconds after the first ball. If both balls reach the ground simultaneously after 5 seconds of dropping the first ball, then the difference between the initial heights of the two balls will be: ( $g=9.8 \text{ m/s}^2$ )  
(1) 58.8 m (2) 78.4 m  
(3) 98.0 m (4) 117.6 m
61. Drops of water fall from the roof of a building 18m high at regular intervals of time. When the first drop reaches the ground, at the same instant fourth drop begins to fall. What are the distances of the second and third drops from the roof ?  
(1) 6 m and 2 m (2) 6 m and 3 m  
(3) 8 m and 2 m (4) 4 m and 2 m
62. If an iron ball and a wooden ball of same radii are released from a height  $h$  in vacuum then time taken by both of them to reach ground will be :  
(1) unequal (2) exactly equal  
(3) roughly equal (4) zero
63. Water drops fall at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant ?  
(1) 1.25 m (2) 2.50 m  
(3) 3.75 m (4) 4.00 m
64. If a ball is thrown vertically upwards with  $40 \text{ m/s}$ . its velocity after two seconds will be :  
(1)  $10 \text{ m/s}$  (2)  $20 \text{ m/s}$   
(3)  $30 \text{ m/s}$  (4)  $40 \text{ m/s}$
65. A stone is dropped from a certain height which can reach the ground in 5 seconds. It is stopped after 3 seconds of its fall and is again released. The total time taken by the stone to reach the ground will be :  
(1) 6 s (2) 6.5 s  
(3) 7 s (4) 7.5 s
66. With what speed should a body be thrown upwards so that the distances traversed in 5th second and 6th second are equal ?  
(1)  $58.4 \text{ m/s}$  (2)  $49 \text{ m/s}$   
(3)  $\sqrt{98} \text{ m/s}$  (4)  $98 \text{ m/s}$
67. Which of the following four statements is false?  
(1) A body can have zero velocity and still be accelerated  
(2) A body can have a constant velocity and still have a varying speed  
(3) A body can have a constant speed and still have a varying velocity  
(4) The direction of the velocity of a body can change when its acceleration is constant.
68. A body dropped from a tower reaches the ground in 4s. The height of the tower is about :  
(1) 80 m (2) 20 m  
(3) 160 m (4) 40 m



- 69.** A particle is dropped from a certain height. The time taken by it to fall through successive distances of 1 km each will be :

- (1) all equal, being equal to  $\sqrt{\frac{2}{g}}$  second.  
 (2) in the ratio of the square roots of the integers  
 $1 : \sqrt{2} : \sqrt{3}$   
 (3) in the ratio of the difference in the square roots of the integers, i.e.,  
 $\sqrt{1}, (\sqrt{2} - \sqrt{1}), (\sqrt{3} - \sqrt{2}), (\sqrt{4} - \sqrt{3}) \dots\dots$   
 (4) in the ratio of the reciprocals of the square roots of the integers, i.e.,  $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \dots\dots$

- 70.** A ball is thrown upward with a velocity of 100 m/s. It will reach the ground after :-

- (1) 10 s (2) 20 s  
 (3) 5 s (4) 40 s

- 71.** A particle is thrown vertically upward. Its velocity at half of the maximum height is 10m/s. The maximum height attained by it is ( $g = 10 \text{ ms}^{-2}$ ) :-

- (1) 8m (2) 20m  
 (3) 10m (4) 16m

- 72.** Three different objects of masses  $m_1$ ,  $m_2$  and  $m_3$  are allowed to fall from rest and from the same point 'O' along three different frictionless paths. The speeds of the three objects on reaching the ground, will be in the ratio of :-

- (1)  $m_1 : m_2 : m_3$  (2)  $m_1 : 2m_2 : 3m_3$   
 (3)  $1 : 1 : 1$  (4)  $\frac{1}{m_1} : \frac{1}{m_2} : \frac{1}{m_3}$

- 73.** If a ball is thrown vertically upwards with speed  $u$ , the distance covered during the last 't' seconds of its ascent is :-

- (1)  $ut$  (2)  $\frac{1}{2}gt^2$   
 (3)  $ut - \frac{1}{2}gt^2$  (4)  $(u + gt)t$

- 74.** A stone falls freely such that the distance covered by it in the last second of its motion is equal to the distance covered by it in the first 5 seconds. It remained in air for :-

- (1) 12 s (2) 13 s  
 (3) 25 s (4) 26 s

- 75.** When a ball is thrown vertically up with velocity  $v_0$ , it reaches a maximum height 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity -

- (1)  $\sqrt{3} v_0$  (2)  $3v_0$   
 (3)  $9v_0$  (4)  $3/2v_0$

- 76.** An object is dropped vertically down on earth. The change in its speed after falling through a distance  $d$  from its highest point is

- (1)  $mgd$  (2)  $\sqrt{2gd}$   
 (3)  $2\sqrt{gd}$  (4)  $2\sqrt{\frac{mg}{d}}$

- 77.** The ratio of the distances traversed, in successive intervals of time by a body falling from rest, are

- (1)  $1 : 3 : 5 : 7 : 9 : \dots\dots\dots$   
 (2)  $2 : 4 : 6 : 8 : 10 : \dots\dots\dots$   
 (3)  $1 : 4 : 7 : 10 : 13 : \dots\dots\dots$   
 (4) None of these

- 78.** A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second ?

- (1)  $\frac{7}{5}$  (2)  $\frac{5}{7}$   
 (3)  $\frac{7}{3}$  (4)  $\frac{3}{7}$

- 79.** A particle is thrown up vertically with a speed ' $v_1$ ', in air. It takes time  $t_1$  in upward journey and  $t_2$  ( $> t_1$ ) in the downward journey and returns to the starting point with a speed  $v_2$ . Then:

- (1)  $v_1 = v_2$  (2)  $v_1 < v_2$   
 (3)  $v_1 > v_2$  (4) Data is insufficient

- 80.** A ball is thrown vertically upwards. Assuming the air resistance to be constant and considerable :-

- (1) the time of ascent  $\geq$  the time of descent  
 (2) the time of ascent  $<$  the time of descent  
 (3) the time of ascent  $>$  the time of descent  
 (4) the time of ascent = the time of descent

- 81.** A body is projected vertically up at  $t = 0$  with a velocity of 98 m/s. Another body is projected from the same point with same velocity after 4 seconds. Both bodies will meet at  $t =$

- (1) 6 s (2) 8 s  
 (3) 10 s (4) 12 s



82. A body released from a height falls freely towards earth. Another body is released from the same point exactly one second later. The separation between them two seconds after the release of the second body is :-

(1) 9.8 m (2) 49 m  
(3) 24.5 m (4) 19.6 m

83. A particle starts from rest with constant acceleration. The ratio of space-average velocity to the time average velocity is :

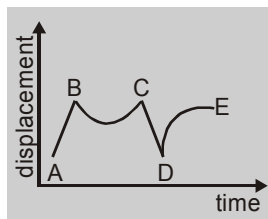
(1)  $\frac{1}{2}$  (2)  $\frac{3}{4}$  (3)  $\frac{4}{3}$  (4)  $\frac{3}{2}$

### GRAPHICAL ANALYSIS

84. Velocity-time curve for a body projected vertically upwards is a/an :-

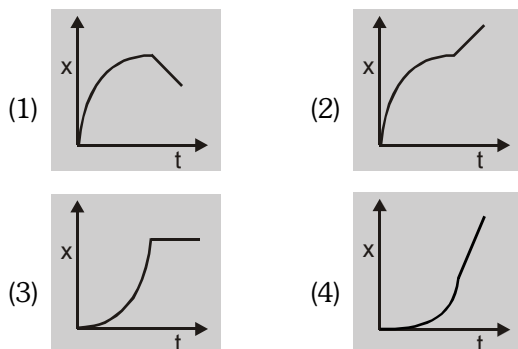
(1) Parabola (2) Ellipse  
(3) Hyperbola (4) Straight line

85. Fig. shows the displacement of a particle moving along x-axis as a function of time. The acceleration of the particle is zero in the region :

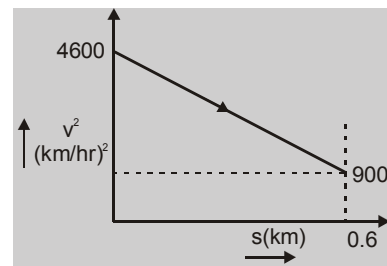


- (a) AB (b) BC  
(c) CD (d) DE  
Select correct alternative  
(1) a, b (2) a, c  
(3) b, d (4) c, d

86. A car starts from rest and accelerates uniformly for 4 seconds and then moves with uniform velocity. Which of the x-t graphs represent the motion of the car ?

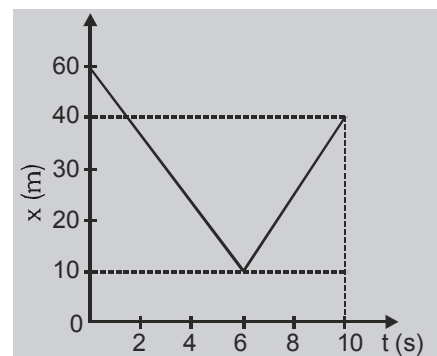


87. Graph between the square of the velocity ( $v$ ) of a particle and the distance ( $s$ ) moved is shown in figure. The acceleration of the particle in kilometers per hour square is :



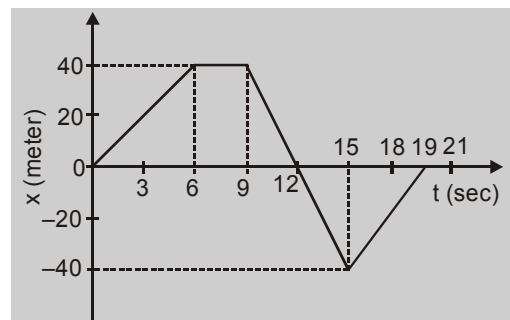
- (1) 2250 (2) 3084  
(3) - 2250 (4) - 3084

88. The fig. shows the position time graph of a particle moving on a straight line path. What is the magnitude of average velocity of the particle over 10 seconds ?



- (1) 2 m/s (2) 4 m/s  
(3) 6 m/s (4) 8 m/s

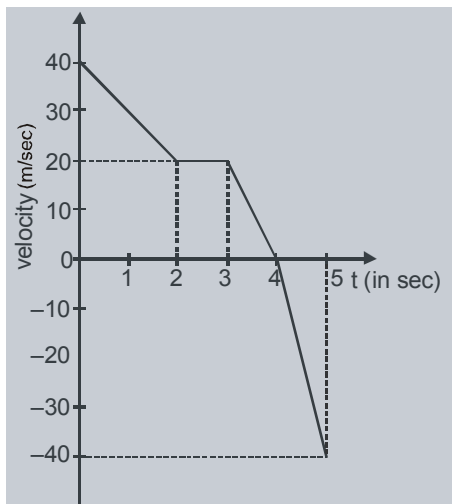
89. A person walks along an east-west street and a graph of his displacement from home is shown in figure. His average velocity for the whole time interval is :



- (1) 0 (2) 23 m/s  
(3) 8.4 m/s (4) None of above

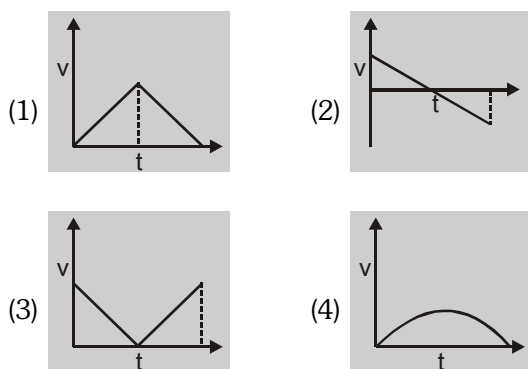


90. From the following velocity time graph of a body the distance travelled by the body and its displacement during 5 seconds in metres will be :

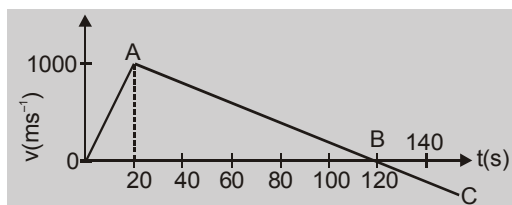


- (1) 75, 75                      (2) 110, 70  
(3) 110, 110                  (4) 110, 40

91. A body is projected vertically upward from the surface of the earth, its velocity-time graph is :



92. A rocket is launched upward from the earth is surface whose velocity time graphs shown in figure. Then maximum height attained by the rocket is :



- (1) 1 km                      (2) 10 km  
(3) 100 km                  (4) 60 km

93. In above question, height covered by the rocket before retardation is :

- (1) 1 km                      (2) 10 km  
(3) 20 km                    (4) 60 km

94. In above question mean velocity of rocket during the time it took to attain the maximum height :

- (1) 100 m/s                      (2) 50 m/s  
(3) 500 m/s                    (4)  $25/3$  m/s

95. In above question the retardation of rocket is :

- (1)  $50 \text{ m/s}^2$                       (2)  $100 \text{ m/s}^2$   
(3)  $500 \text{ m/s}^2$                     (4)  $10 \text{ m/s}^2$

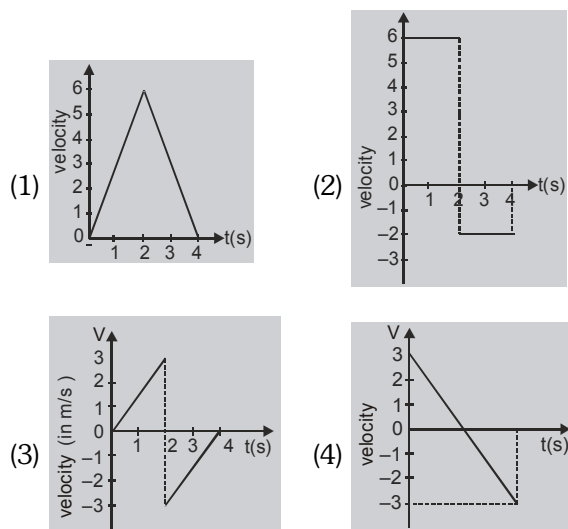
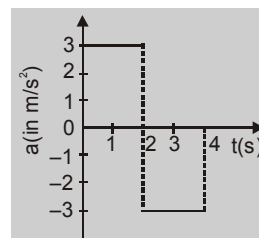
96. In above question the acceleration of rocket is :

- (1)  $50 \text{ m/s}^2$                       (2)  $100 \text{ m/s}^2$   
(3)  $10 \text{ m/s}^2$                     (4)  $1000 \text{ m/s}^2$

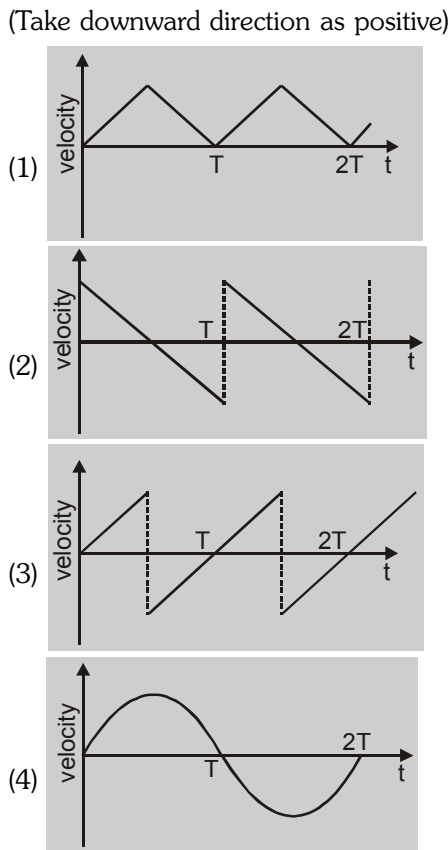
97. In above question the rocket goes up and comes down on the following parts respectively :

- (1) OA and AB  
(2) AB and BC  
(3) OA and ABC  
(4) OAB and BC

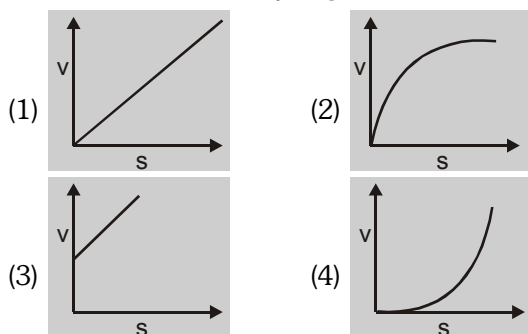
98. For the motion of a particle acceleration-time graph is shown in figure. The velocity time curve for the duration of 0 – 4 seconds is :



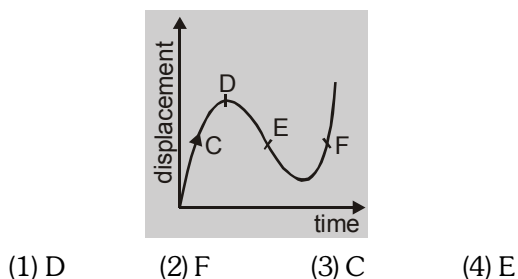
- 99.** A ball is dropped from the certain height on the surface of glass. It collides elastically and comes back to its initial position. If this process it repeated then the velocity time graph is :



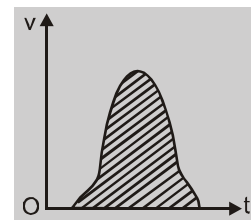
- 100.** A particle starts from rest and move with constant acceleration. Its velocity-displacement curve is :



- 101.** The displacement-time graph of a moving particle is shown. The instantaneous velocity of the particle is negative at the point :

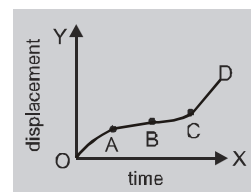


- 102.** Figure below shows the velocity-time graph of a one dimensional motion. Which of the following characteristics of the particle is represented by the shaded area?



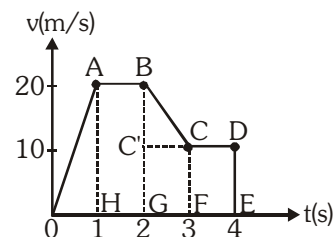
- (1) Speed (2) Displacement  
(3) Acceleration (4) Momentum

- 103.** The graph between the displacement  $x$  and time  $t$  for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is :



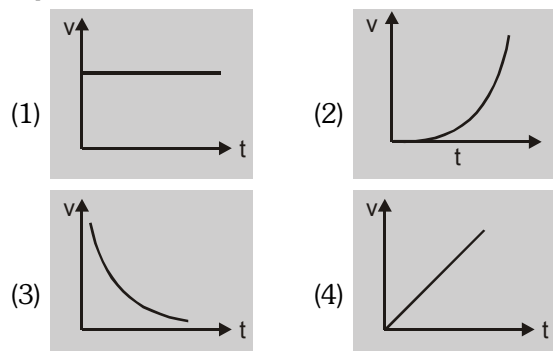
	OA	AB	BC	CD
(1)	+	0	+	+
(2)	-	0	+	0
(3)	+	0	-	+
(4)	-	0	-	0

- 104.** The variation of velocity of a particle moving along a straight line is illustrated in the figure. The distance transversed by the particle in 4 seconds is

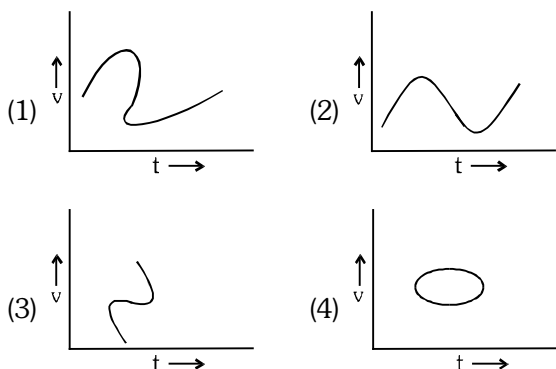


- (1) 60 m (2) 25 m (3) 55 m (4) 30 m

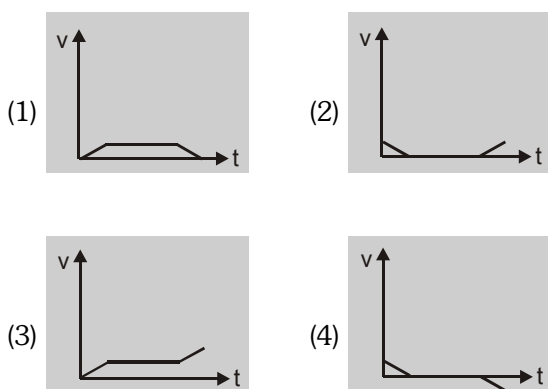
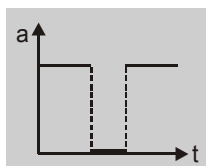
- 105.** Which of the following velocity-time graphs represent uniform motion ?



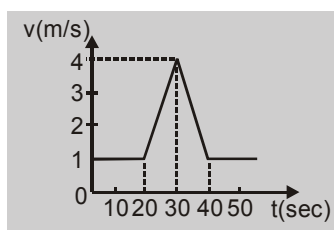
106. Which of the following velocity-time graphs shows a realistic situation for a body in motion ?



107. Acceleration-time graph of a body is shown. The corresponding velocity-time graph is :

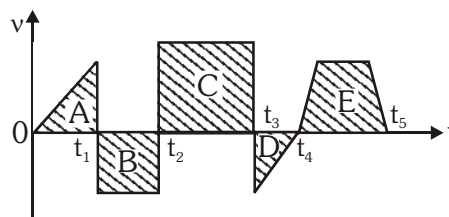


108. Velocity-time (v-t) graph for a moving object is shown in the figure. Total displacement of the object during the time interval when there is non-zero acceleration and retardation is :-



- (1) 60 m (2) 50 m  
(3) 30 m (4) 40 m

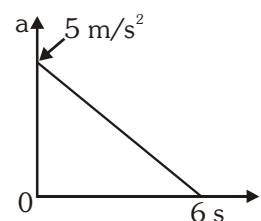
109. The velocity-time graph of an object is shown. The displacement during the interval 0 to  $t_4$  is :-



- (1) Area A + Area B + Area C + Area D + Area E  
(2) Area A + Area C - Area B - Area D  
(3) Area A + Area B + Area C + Area D  
(4) Area A + Area C + Area E - Area B + Area D

110. A particle starts from rest. Its acceleration at time  $t = 0$  is  $5 \text{ m/s}^2$  which varies with time as shown in the figure. The maximum speed of the particle will be :

- (1) 7.5 m/s  
(2) 15 m/s  
(3) 30 m/s  
(4) 37.5 m/s

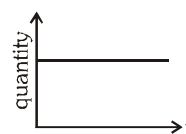


111. A car accelerates from rest at a constant rate of  $2 \text{ m/s}^2$  for some time. Then, it retards at a constant rate of  $4 \text{ m/s}^2$  and comes to rest. If it remains in motion for 3 seconds, then the maximum speed attained by the car is :
- (1) 2 m/s (2) 3 m/s  
(3) 4 m/s (4) 6 m/s

### GROUND TO GROUND PROJECTION

112. In the graph shown in fig. time is plotted along x-axis. Which quantity associated with a projectile motion is plotted along the y-axis ?

- (1) kinetic energy  
(2) momentum  
(3) horizontal velocity  
(4) none of the above



113. In case of a projectile fired at an angle equally inclined to the horizontal and vertical with velocity  $u$ , the horizontal range is :

- (1)  $\frac{u^2}{g}$  (2)  $\frac{u^2}{2g}$   
(3)  $\frac{2u^2}{g}$  (4)  $\frac{u^2}{g^2}$



- 114.** A shell is fired vertically upwards with a velocity  $v_1$  from the deck of a ship moving with a speed  $v_2$ . A person on the shore observes the motion of the shell as a parabola. Its horizontal range is given by :

(1)  $\frac{2v_1^2 v_2}{g}$  (2)  $\frac{2v_1 v_2^2}{g}$   
 (3)  $\frac{2v_1 v_2}{g}$  (4)  $\frac{2v_1^2 v_2^2}{g}$

- 115.** The range of a projectile when fired at  $75^\circ$  to the horizontal is 0.5 km. What will be its range when fired at  $45^\circ$  with the same speed ?

(1) 0.5 km (2) 1.0 km  
 (3) 1.5 km (4) 2.0 km

- 116.** A particle is projected with a velocity  $u$  making an angle  $\theta$  with the horizontal. At any instant, its velocity  $v$  is at right angle to its initial velocity  $u$ ; then  $v$  is:

(1)  $u \cos \theta$  (2)  $u \tan \theta$   
 (3)  $u \cot \theta$  (4)  $u \sec \theta$

- 117.** The speed of a projectile at its maximum height is  $\frac{\sqrt{3}}{2}$  times of its initial speed ' $u$ ' of projection.

Its range on the horizontal plane is :

(1)  $\frac{\sqrt{3}u^2}{2g}$  (2)  $\frac{u^2}{2g}$   
 (3)  $\frac{3u^2}{2g}$  (4)  $\frac{3u^2}{g}$

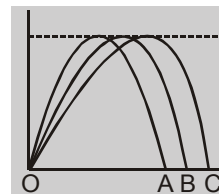
- 118.** A ball is thrown at an angle  $\theta$  to the horizontal and the range is maximum. The value of  $\tan \theta$  is :

(1) 1 (2)  $\sqrt{3}$   
 (3)  $\frac{1}{\sqrt{3}}$  (4) 2

- 119.** A student is able to throw a ball vertically to a maximum height of 40 m. The maximum distance to which he can throw the ball in the horizontal direction is :

(1) 40 (2)  $20^{1/2}$  m (2)  $20(2)^{1/2}$  m  
 (3) 20 m (4) 80 m

- 120.** Three projectiles A, B and C are thrown from the same point in the same plane. Their trajectories are shown in the figure. Which of the following statement is true ?



- (1) The time of flight is the same for all the three  
 (2) The launch speed is largest for particle C  
 (3) The horizontal velocity component is largest for particle C  
 (4) All of the above

- 121.** A projectile is thrown with an initial velocity of  $\vec{v} = a\hat{i} + b\hat{j}$ . If range of the projectile is double the maximum height attained by it then :

(1)  $a = 2b$  (2)  $b = a$   
 (3)  $b = 2a$  (4)  $b = 4a$

- 122.** The equation of a projectile is  $y = \sqrt{3}x - \frac{gx^2}{2}$ . The angle of projection is :

(1)  $30^\circ$  (2)  $60^\circ$  (3)  $45^\circ$  (4) none

- 123.** The equation of a projectile is  $y = 16x - \frac{x^2}{4}$ . The horizontal range is :

(1) 16 m (2) 8 m (3) 64 m (4) 12.8 m

- 124.** If a projectile is fired at an angle  $\theta$  to the vertical with velocity  $u$ , then maximum height attained is given by :

(1)  $\frac{u^2 \cos \theta}{2g}$  (2)  $\frac{u^2 \sin^2 \theta}{2g}$   
 (3)  $\frac{u^2 \sin^2 \theta}{g}$  (4)  $\frac{u^2 \cos^2 \theta}{2g}$

- 125.** If  $R$  is the maximum horizontal range of a particle, then the greatest height attained by it is :

(1)  $R$  (2)  $2R$  (3)  $\frac{R}{2}$  (4)  $\frac{R}{4}$

- 126.** Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one

is  $\frac{\pi}{3}$  and its maximum height is  $y_1$ , then the maximum height of the other will be :

(1)  $3y_1$  (2)  $2y_1$  (3)  $\frac{y_1}{2}$  (4)  $\frac{y_1}{3}$



**127.** A projectile is thrown from a point in a horizontal plane such that its horizontal and vertical velocity components are 9.8 m/s and 19.6 m/s respectively. Its horizontal range is :

- (1) 4.9 m (2) 9.8 m  
(3) 19.6 m (4) 39.2 m

**128.** A projectile is thrown into space so as to have the maximum possible horizontal range equal to 400m. Taking the point of projection as the origin, the coordinates of the point where the velocity of the projectile is minimum are:

- (1) (400, 100) m  
(2) (200, 100) m  
(3) (400, 200) m  
(4) (200, 200) m

**129.** A particle is fired with velocity  $u$  making  $\theta$  angle with the horizontal. What is the change in velocity when it is at the highest point ?

- (1)  $u \cos \theta$  (2)  $u$   
(3)  $u \sin \theta$  (4)  $(u \cos \theta - u)$

**130.** In the above, the change in speed is :

- (1)  $u \cos \theta$  (2)  $u$   
(3)  $u \sin \theta$  (4)  $(u \cos \theta - u)$

**131.** An arrow is shot into the air. Its range is 200 metres and its time of flight is 5 s. If the value of  $g$  is assumed to be  $10 \text{ m/s}^2$ , then the horizontal component of the velocity of arrow is :

- (1) 25 m/s (2) 40 m/s  
(3) 31.25 m/s (4) 12.5 m/s

**132.** In the Q. 133, the maximum height attained by the arrow is :

- (1) 25 m (2) 40 m  
(3) 31.25 m (4) 12.5 m

**133.** In the Q. 133, the vertical component of the velocity is :

- (1) 25 m/s (2) 40 m/s  
(3) 12.5 m/s (4) 31.25 m/s

**134.** In the Q. 133, the angle of projection with the horizontal is :

- (1)  $\tan^{-1}\left(\frac{4}{5}\right)$  (2)  $\tan^{-1}\left(\frac{5}{4}\right)$   
(3)  $\tan^{-1}\left(\frac{5}{8}\right)$  (4)  $\tan^{-1}\left(\frac{8}{5}\right)$

**135.** A ball is thrown at different angles with the same speed  $u$  and from the same point; it has the same range in both the cases. If  $y_1$  and  $y_2$  be the heights attained in the two cases, then  $y_1 + y_2 = \dots$  :

- (1)  $\frac{u^2}{g}$  (2)  $\frac{2u^2}{g}$   
(3)  $\frac{u^2}{2g}$  (4)  $\frac{u^2}{4g}$

**136.** Two balls A and B are thrown with speeds  $u$  and  $u/2$  respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is  $15^\circ$  with the horizontal, then the angle of projection of A is :

- (1)  $\sin^{-1}\left(\frac{1}{8}\right)$  (2)  $\frac{1}{2} \sin^{-1}\left(\frac{1}{8}\right)$   
(3)  $\frac{1}{3} \sin^{-1}\left(\frac{1}{8}\right)$  (4)  $\frac{1}{4} \sin^{-1}\left(\frac{1}{8}\right)$

**137.** At what angle to the horizontal should a ball be thrown so that its range  $R$  is related to the time of flight  $T$  as  $R = 5T^2$ ? Take  $g = 10 \text{ ms}^{-2}$  :

- (1)  $30^\circ$  (2)  $45^\circ$   
(3)  $60^\circ$  (4)  $90^\circ$

**138.** A projectile is projected with initial velocity  $(6\hat{i} + 8\hat{j}) \text{ m/s}$ . If  $g = 10 \text{ ms}^{-2}$ , then horizontal range is :

- (1) 4.8 metre (2) 9.6 metre  
(3) 19.2 metre (4) 14.0 metre

**139.** If the range of a gun which fires a shell with muzzle speed  $v$ , is  $R$ , then the angle of elevation of the gun is :

- (1)  $\cos^{-1}\left(\frac{v^2}{Rg}\right)$  (2)  $\cos^{-1}\left(\frac{Rg}{v^2}\right)$   
(3)  $\frac{1}{2} \sin^{-1}\left(\frac{v^2}{Rg}\right)$  (4)  $\frac{1}{2} \sin^{-1}\left(\frac{Rg}{v^2}\right)$

**140.** The maximum range of a projectile fired with some initial velocity is found to be 1000 m. The maximum height ( $H$ ) reached by this projectile is :

- (1) 250 metre (2) 500 metre  
(3) 1000 metre (4) 2000 metre



- 141.** The angle which the velocity vector of a projectile, will make with the horizontal after time  $t$  of its being thrown with a velocity  $v$  at an angle  $\theta$  to the horizontal, is :
- (1)  $\theta$  (2)  $\tan^{-1}\left(\frac{\theta}{t}\right)$   
 (3)  $\tan^{-1}\left(\frac{v \cos \theta}{v \sin \theta - gt}\right)$  (4)  $\tan^{-1}\left(\frac{v \sin \theta - gt}{v \cos \theta}\right)$
- 142.** A particle is projected at an angle of  $45^\circ$ , 8m away from the foot of a wall, just touches the top of the wall and falls on the ground on the opposite side at a distance 4m from it. The height of wall is :
- (1)  $\frac{2}{3}$  m (2)  $\frac{4}{3}$  m (3)  $\frac{8}{3}$  m (4)  $\frac{3}{4}$  m
- 143.** The maximum horizontal range of a gun is 16 km. If  $g = 10 \text{ m/s}^2$ , the muzzle velocity of the shell must be :-
- (1) 1600 m/s (2) 400 m/s  
 (3)  $200\sqrt{2}$  m/s (4)  $160\sqrt{10}$  m/s
- 144.** A body is thrown with a velocity of 9.8 m/s making an angle of  $30^\circ$  with the horizontal. It will hit the ground after a time :-
- (1) 3 s (2) 2 s  
 (3) 1.5 s (4) 1 s
- 145.** Three particles A, B and C are projected from the same point with the same initial speeds making angles  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  respectively with the horizontal. Which of the following statements are correct ?
- (1) A, B and C have unequal ranges  
 (2) Ranges of A and C are equal and less than that of B  
 (3) Ranges of A and C are equal and greater than that of B  
 (4) A, B and C have equal ranges
- 146.** A ball whose kinetic energy is  $E$ , is thrown at an angle of  $45^\circ$  to the horizontal. Its kinetic energy at the highest point of its flight will be :-
- (1)  $E$  (2)  $\frac{E}{2}$  (3)  $\frac{E}{2}$  (4) zero
- 147.** A body is projected at such an angle that the horizontal range is three times the greatest height. The angle of projection is :-
- (1)  $25^\circ$  (2)  $33^\circ$  (3)  $42^\circ$  (4)  $53^\circ$
- 148.** A projectile can have the same range  $R$  for two angles of projection. If  $t_1$  and  $t_2$  be the times of flight in the two cases, then :-
- (1)  $t_1 t_2 \propto R^2$  (2)  $t_1 t_2 \propto R$   
 (3)  $t_1 t_2 \propto \frac{1}{R}$  (4)  $t_1 t_2 \propto \frac{1}{R^2}$
- 149.** A body is thrown with some velocity from the ground. Maximum height attained when it is thrown at  $60^\circ$  to the horizontal is 90 m. What is the height attained when it is thrown at  $30^\circ$  to the horizontal ?
- (1) 90 m (2) 45 m  
 (3) 30 m (4) 15 m
- 150.** At the uppermost point of a projectile its velocity and acceleration are at an angle of :-
- (1)  $180^\circ$  (2)  $90^\circ$   
 (3)  $60^\circ$  (4)  $45^\circ$
- 151.** A force  $\vec{F} = 6t^2\hat{i} + 4t\hat{j}$  acts on a particle of mass 3 kg. What will be velocity of the particle at  $t = 3$  second if at  $t = 0$ , the particle was at rest :-
- (1)  $18\hat{i} + 6\hat{j}$  (2)  $18\hat{i} + 12\hat{j}$   
 (3)  $12\hat{i} + 6\hat{j}$  (4) none
- 152.** A number of bullets are fired in all possible directions with the same initial velocity  $u$ . The maximum area of ground covered by bullets is :-
- (1)  $\pi\left(\frac{2u^2}{g}\right)^2$  (2)  $3\pi\left(\frac{u}{g}\right)^2$   
 (3)  $5\pi\left(\frac{u}{2g}\right)^2$  (4)  $\pi\left(\frac{u^2}{g}\right)^2$
- 153.** For a given angle of projection if the initial velocity is doubled the range of the projectile becomes :-
- (1) half (2) one-fourth  
 (3) two times (4) four times
- 154.** A ball is projected to attain the maximum range. If the height attained is  $H$ , the range is
- (1)  $H$  (2)  $2H$   
 (3)  $4H$  (4)  $H/2$



**155.** Two projectiles are fired from the same point with the same speeds at angles of projection  $60^\circ$  and  $30^\circ$  respectively. Which one of the following is true ?

- (1) Their horizontal ranges will be the same
- (2) Their maximum heights will be the same
- (3) Their landing velocities will be the same
- (4) Their times of flight will be the same

**156.** A ball is projected vertically upwards with a certain speed. Another ball of the same mass is projected at an angle  $60^\circ$  to the vertical with the same initial speed. The ratio of their potential energies at highest points of their journey, will be  
(1) 1 : 1      (2) 2 : 1      (3) 3 : 2      (4) 4 : 1

#### PROJECTION FROM A HEIGHT

**157.** A body is thrown horizontally with a velocity  $\sqrt{2gh}$  from the top of a tower of height  $h$ . It strikes the ground level through the foot of the tower at a distance  $x$  from the tower. The value of  $x$  is :

- (1)  $h$       (2)  $\frac{h}{2}$       (3)  $2h$       (4)  $\frac{3}{4}h$

**158.** When a particle is thrown horizontally, the resultant velocity of the projectile at any time  $t$  is given by :

- (1)  $gt$       (2)  $\frac{1}{2}gt^2$
- (3)  $\sqrt{u^2 + g^2t^2}$       (4)  $\sqrt{u^2 - g^2t^2}$

**159.** A ball is projected upwards from the top of a tower with a velocity of  $50 \text{ m/s}$  making an angle of  $30^\circ$  with the horizontal. The height of the tower is  $70 \text{ m}$ . After how much time from the instant of throwing, will the ball reach the ground ?

- (1)  $2 \text{ s}$       (2)  $5 \text{ s}$
- (3)  $7 \text{ s}$       (4)  $9 \text{ s}$

**160.** An aeroplane moving horizontally with a speed of  $180 \text{ km/h}$  drops a food packet while flying at a height of  $490 \text{ m}$ . The horizontal range of the packet is :

- (1)  $180 \text{ m}$       (2)  $980 \text{ m}$
- (3)  $500 \text{ m}$       (4)  $670 \text{ m}$

**161.** A plane is flying horizontally at  $98 \text{ m/s}$  and releases an object which reaches the ground in  $10 \text{ s}$ . The angle made by it while hitting the ground is :

- (1)  $55^\circ$       (2)  $45^\circ$
- (3)  $60^\circ$       (4)  $75^\circ$

**162.** A stuntman plans to run along a roof top and then horizontally off it to land on the roof of next building. The roof of the next building is  $4.9 \text{ metres}$  below the first one and  $6.2 \text{ metres}$  away from it. What should be his minimum roof top speed in  $\text{m/s}$ , so that he can successfully make the jump ?

- (1)  $3.1$       (2)  $4.0$
- (3)  $4.9$       (4)  $6.2$

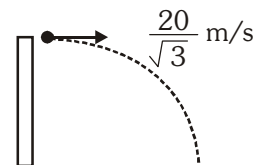
**163.** From the top of a tower  $19.6 \text{ m}$  high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of  $45^\circ$  with the horizontal, then the initial velocity of the ball is :

- (1)  $9.8 \text{ m/s}$       (2)  $4.9 \text{ m/s}$
- (3)  $14.7 \text{ m/s}$       (4)  $2.8 \text{ m/s}$

**164.** A bomber is flying horizontally with a constant speed of  $150 \text{ m/s}$  at a height of  $78.4 \text{ m}$ . The pilot has to drop a bomb at the enemy target. At what horizontal distance from the target should he release the bomb?

- (1)  $0 \text{ m}$       (2)  $300 \text{ m}$
- (3)  $600 \text{ m}$       (4)  $1000 \text{ m}$

**165.** A particle is projected horizontally with a speed of  $\frac{20}{\sqrt{3}} \text{ m/s}$ , from some height at  $t = 0$ . At what time will its velocity make  $60^\circ$  angle with the initial velocity



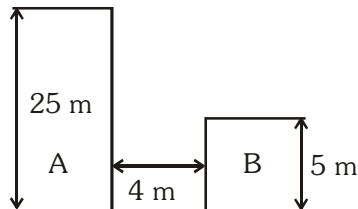
- (1)  $1 \text{ sec}$       (2)  $2 \text{ sec}$       (3)  $1.5 \text{ sec}$       (4)  $2.5 \text{ sec}$

**166.** In the above question what will be the displacement of the particle in  $x$ -direction when its velocity makes  $60^\circ$  angle with the initial velocity

- (1)  $\frac{20}{\sqrt{3}} \text{ m}$       (2)  $\frac{40}{\sqrt{3}} \text{ m}$       (3)  $\frac{50}{\sqrt{3}}$       (4)  $\frac{10}{\sqrt{3}}$



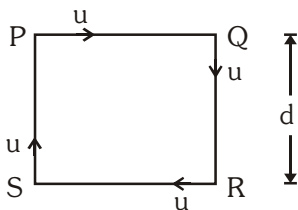
- 167.** A boy wants to jump from building A to building B. Height of building A is 25 m and that of building B is 5m. Distance between buildings is 4m. Assume that the boy jumps horizontally, then calculate minimum velocity with which he has to jump to land safely on building B.



- (1) 6 m/s    (2) 8 m/s    (3) 4 m/s    (4) 2 m/s

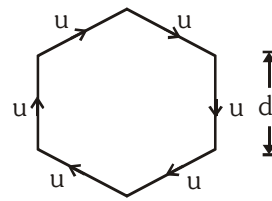
### RELATIVE MOTION IN ONE DIMENSION

- 168.** A train moves in north direction with a speed of 54 km/h. A monkey is running on the roof of the train, against its motion with a velocity of 18 km/h. with respect to train. The velocity of monkey as observed by a man standing on the ground is:  
 (1) 5 m/s due south    (2) 25 m/s due south  
 (3) 10 m/s due south    (4) 10 m/s due north
- 169.** A lift is moving downwards with acceleration  $a$ . A man in the lift drops a ball within the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively :  
 (1)  $g, g$     (2)  $g - a, g - a$   
 (3)  $g - a, g$     (4)  $a, g$
- 170.** A boat takes 2 hours to go 8 km and come back in still water lake. The time taken for going 8 km upstream and coming back with water velocity of 4 km/h is :  
 (1) 140 min    (2) 150 min  
 (3) 160 min    (4) 170 min
- 171.** Four persons P, Q, R and S of same mass travel with same speed  $u$  along a square of side ' $d$ ' such that each one always faces the other. After what time will they meet each other ?



- (1)  $\frac{d}{u}$     (2)  $\frac{2d}{3u}$     (3)  $\frac{2d}{u}$     (4)  $d\sqrt{3}u$

- 172.** Six persons of same mass travel with same speed  $u$  along a regular hexagon of side ' $d$ ' such that each one always faces the other. After how long will they meet each other ?



- (1)  $\frac{d}{u}$     (2)  $\frac{2d}{3u}$   
 (3)  $\frac{2d}{u}$     (4)  $d\sqrt{3}u$

- 173.** A person walks up a stalled escalator in 90 sec. He is carried in 60s, when standing on the same escalator which is now moving. The time he would take to walk up the moving escalator will be :-  
 (1) 27 s    (2) 72 s  
 (3) 18 s    (4) 36 s
- 174.** A jet air plane travelling with a speed of 500 km/h ejects its products of combustion with a speed of 1500 km/h relative to the jet plane. The speed of the latter with respect to an observer on the ground is :-  
 (1) 1500 km/h    (2) 2000 km/h  
 (3) 1000 km/h    (4) 500 km/h
- 175.** A train of 150 m length is running towards north at a speed of 10 m/s. A parrot flies at a speed of 5 m/s towards south parallel to the railway track. The time taken by the parrot to cross the train is equal to :-  
 (1) 12 s    (2) 8 s  
 (3) 15 s    (4) 10 s
- 176.** Two trains each 50m long, are travelling in opposite directions with respective velocities 10 m/s and 15 m/s. The time of crossing is :-  
 (1) 2 s    (2) 4 s  
 (3)  $2\sqrt{3}$  s    (4)  $4\sqrt{3}$  s
- 177.** Two cars are moving in the same direction with the same speed of 30 km/h. They are separated by 5 km. What is the speed of a car moving in the opposite direction if it meets the two cars at an interval of 4 minute ?  
 (1) 45 km/h    (2) 60 km/h  
 (3) 105 km/h    (4) None



- 178.** A stone is thrown upwards and it rises to a height of 200 m. The relative velocity of the stone with respect to the earth will be maximum at :-  
 (1) Height of 100 m (2) Height of 150 m  
 (3) Highest point (4) The ground
- 179.** A bus starts from rest moving with an acceleration of  $2\text{ m/s}^2$ . A cyclist, 96 m behind the bus starts simultaneously towards the bus at  $20\text{ m/s}$ . After what time will he be able to overtake the bus :-  
 (1) 8 s (2) 10 s (3) 12 s (4) 1 s
- 180.** A train is moving towards East with a speed  $20\text{ m/s}$ . A person is running on the roof of the train with a speed  $3\text{ m/s}$  against the motion of train. Velocity of the person as seen by an observer on ground will be :  
 (1)  $23\text{ m/s}$  towards East  
 (2)  $17\text{ m/s}$  towards East  
 (3)  $23\text{ m/s}$  towards West  
 (4)  $17\text{ m/s}$  towards West
- 181.** A motorcycle is moving with a velocity of  $80\text{ km/h}$  ahead of a car moving with a velocity of  $65\text{ km/h}$  in the same direction. What is the relative velocity of the motorcycle with respect to the car ?  
 (1)  $15\text{ km/h}$  (2)  $20\text{ km/h}$   
 (3)  $25\text{ km/h}$  (4)  $145\text{ km/h}$ .
- 182.** A  $100\text{ m}$  long train crosses a man travelling at  $5\text{ km/h}$ , in opposite direction in  $7.2$  seconds, then the velocity of train is :-  
 (1)  $40\text{ km/h}$  (2)  $25\text{ km/h}$   
 (3)  $20\text{ km/h}$  (4)  $45\text{ km/h}$
- 183.** An elevator is accelerating upward at a rate of  $6\text{ ft/sec}^2$  when a bolt from its ceiling falls to the floor of the lift (Distance =  $9.5$  feet). The time (in seconds) taken by the falling bolt to hit the floor is (take  $g = 32\text{ ft/sec}^2$ )  
 (1)  $\sqrt{2}$  (2)  $\frac{1}{\sqrt{2}}$   
 (3)  $2\sqrt{2}$  (4)  $\frac{1}{2\sqrt{2}}$
- 184.** A  $210$  metres long train is moving due north with a speed of  $25\text{ m/s}$ . A small bird is flying due south a little above the train with  $5\text{ m/s}$  speed. The time taken by the bird to cross the train is :-  
 (1) 6 s (2) 7 s (3) 9 s (4) 10 s

- 185.** Two balls are thrown simultaneously, (A) vertically upwards with a speed of  $20\text{ m/s}$  from the ground and (B) vertically downwards from a height of  $40\text{ m}$  with the same speed and along the same line of motion. At which point will the balls collide? (take  $g = 10\text{ m/s}^2$ )  
 (1)  $15\text{ m}$  above from the ground  
 (2)  $15\text{ m}$  below from the top of the tower  
 (3)  $20\text{ m}$  above from the ground  
 (4)  $20\text{ m}$  below from the top of the tower
- 186.** A body A is thrown up vertically from the ground with velocity  $v_0$  and another body B is simultaneously dropped from a height  $H$ . They meet at a height  $\frac{H}{2}$  if  $v_0$  is equal to

(1)  $\sqrt{2gH}$  (2)  $\sqrt{gH}$  (3)  $\frac{1}{2}\sqrt{gH}$  (4)  $\sqrt{\frac{2g}{H}}$

#### RELATIVE MOTION IN TWO DIMENSION

- 187.** A boy is running on a levelled road with velocity ( $v$ ) with a long hollow tube in his hand. Water is falling vertically downwards with velocity ( $u$ ). At what angle to the vertical, should he incline the tube so that the water drops enters without touching its side :  
 (1)  $\tan^{-1}\left(\frac{v}{u}\right)$  (2)  $\sin^{-1}\left(\frac{v}{u}\right)$   
 (3)  $\tan^{-1}\left(\frac{u}{v}\right)$  (4)  $\cos^{-1}\left(\frac{v}{u}\right)$
- 188.** A river flows from east to west with a speed of  $5\text{ m/min}$ . A man on south bank of river, capable of swimming at the rate of  $10\text{ m/min}$  in still water, wants to swim across the river in shortest time; he should swim :  
 (1) due north  
 (2) due north-east  
 (3) due north-east with double the speed of river  
 (4) none of the above
- 189.** A boat is sailing with a velocity  $(3\hat{i} + 4\hat{j})$  with respect to ground and water in river is flowing with a velocity  $(-3\hat{i} - 4\hat{j})$ . Relative velocity of the boat with respect to water is :  
 (1)  $8\hat{j}$  (2)  $5\sqrt{2}$   
 (3)  $6\hat{i} + 8\hat{j}$  (4)  $-6\hat{i} - 8\hat{j}$



**190.** A river is flowing at the rate of 6 km/h. A swimmer swims across the river with a velocity of 9 km/h w.r.t. water. The resultant velocity of the man will be in (km/h):-

- (1)  $\sqrt{117}$  (2)  $\sqrt{340}$  (3)  $\sqrt{17}$  (4)  $3\sqrt{40}$

**191.** A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h in still water and the river flows at the rate of 1 km/h. The angle made by the direction (w.r.t. the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point, should be :

- (1)  $60^\circ$  (2)  $120^\circ$  (3)  $145^\circ$  (4)  $90^\circ$

**192.** A boat-man can row a boat to make it move with a speed of 10 km/h in still water. River flows steadily at the rate of 5 km/h. and the width of the river is 2 km. If the boat man cross the river along the minimum distance of approach then time elapsed in rowing the boat will be :

- (1)  $\frac{2\sqrt{3}}{5}$  h (2)  $\frac{2}{5\sqrt{3}}$  h (3)  $\frac{3\sqrt{2}}{5}$  h (4)  $\frac{5\sqrt{2}}{3}$  h

**193.** A bird is flying towards south with a velocity 40 km/h and a train is moving with a velocity 40 km/h towards east. What is the velocity of the bird w.r.t. an observer in the train ?

- (1)  $40\sqrt{2}$  km/h. N-E  
(2)  $40\sqrt{2}$  km/h. S-E  
(3)  $40\sqrt{2}$  km/h. S-W  
(4)  $40\sqrt{2}$  km/h. N-W

**194.** A bird is flying with a speed of 40 km/h in the north direction. A train is moving with a speed of 40 km/h in the west direction. A passenger sitting in the train will see the bird moving with velocity :

- (1) 40 km/h in N-E direction  
(2)  $40\sqrt{2}$  km/h in N-E direction  
(3) 40 km/h in N-W direction  
(4)  $40\sqrt{2}$  km/h in N-W direction

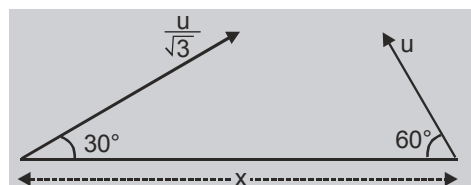
**195.** A particle is moving with a velocity of 10 m/s towards east. After 10 s its velocity changes to 10 m/s towards north. Its average acceleration is :-

- (1) zero  
(2)  $\sqrt{2}$  m/s<sup>2</sup> towards N-W  
(3)  $\frac{1}{\sqrt{2}}$  m/s<sup>2</sup> towards N-E  
(4)  $\frac{1}{\sqrt{2}}$  m/s<sup>2</sup> towards N-W

**196.** A man is walking on a road with a velocity of 3 km/h when suddenly, it starts raining velocity of rain is 10 km/h in vertically downward direction, relative velocity of the rain with respect to man is :

- (1)  $\sqrt{13}$  km/hr (2)  $\sqrt{7}$  km/hr  
(3)  $\sqrt{109}$  km/hr (4) 13 km/h

**197.** Two particles are separated by a horizontal distance  $x$  as shown in figure. They are projected as shown in figure with different initial speeds. The time after which the horizontal distance between them becomes zero is :



- (1)  $\frac{x}{u}$  (2)  $\frac{u}{2x}$   
(3)  $\frac{2u}{x}$  (4) none of these

**198.** Let  $\vec{r}_1(t) = 3t\hat{i} + 4t^2\hat{j}$  and  $\vec{r}_2(t) = 4t^2\hat{i} + 3t\hat{j}$  represent the positions of particles 1 and 2, respectively as functions of time  $t$ ;  $\vec{r}_1(t)$  and  $\vec{r}_2(t)$  are in metres and  $t$  is in seconds. The relative speed of the two particles at the instant  $t = 1$  s, will be

- (1) 1 m/s (2)  $3\sqrt{2}$  m/s  
(3)  $5\sqrt{2}$  m/s (4)  $7\sqrt{2}$  m/s

**199.** A river 4.0 miles wide is flowing at the rate of 2 miles/hr. The minimum time taken by a boat to cross the river with a speed  $v = 4$  miles/hr (in still water) is approximately

- (1) 1 hr and 0 minute  
(2) 2 hr and 7 minutes  
(3) 1 hr and 12 minutes  
(4) 2 hr and 25 minutes

**200.** A river 2 km wide is flows at the rate of 2 km/h. A boatman who can row a boat at a speed of 4 km/h in still water, goes a distance of 2 km upstream and then comes back. The time taken by him to complete his journey is

- (1) 60 min (2) 70 min  
(3) 80 min (4) 90 min



**201.** Two cars A and B start moving from the same point with same speed  $v = 5 \text{ km/minute}$ . Car A moves towards North and car B is moving towards East. What is the relative velocity of B with respect to A?

- (1)  $5\sqrt{2} \text{ km/min}$  towards South-East
- (2)  $5\sqrt{2} \text{ km/min}$  towards North-West
- (3)  $5\sqrt{2} \text{ km/min}$  towards South-West
- (4)  $5\sqrt{2} \text{ km/min}$  towards North-East

### EXERCISE-I (Conceptual Questions)

### ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	3	3	2	1	1	1	4	1	2	1	2	2	2	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	4	3	3	2	3	3	3	2	1	3	4	1	3	3	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	1	4	3	2	2	2	1	3	3	3	2	1	4	2	2
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	4	2	1	4	4	4	2	2	2	3	3	3	2	1	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	3	2	3	2	3	2	2	1	3	2	3	3	2	2	1
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	2	1	1	3	2	4	3	3	4	2	4	4	1	1	2
Que.	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Ans.	2	4	2	3	4	1	4	1	3	2	4	2	2	3	1
Que.	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	2	3	2	2	2	3	3	1	3	2	3	1	1	4	4
Que.	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135
Ans.	3	2	3	4	4	4	4	2	3	4	2	3	1	3	3
Que.	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150
Ans.	2	2	2	4	1	4	3	2	4	2	3	4	2	3	2
Que.	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165
Ans.	1	4	4	3	1	4	3	3	3	3	2	4	1	3	2
Que.	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
Ans.	2	4	4	3	3	1	3	4	3	4	2	1	4	1	2
Que.	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195
Ans.	1	4	2	2	1	2	1	1	3	1	2	2	3	2	2
Que.	196	197	198	199	200	201									
Ans.	3	1	3	1	3	1									



## EXERCISE-II (Assertion & Reason)

### Directions for Assertion & Reason questions

These Questions consist of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

- (A) If both Assertion & Reason are True & the Reason is a correct explanation of the Assertion.  
(B) If both Assertion & Reason are True but Reason is not a correct explanation of the Assertion.  
(C) If Assertion is True but the Reason is False.  
(D) If both Assertion & Reason are false.

1. **Assertion :** A body can have acceleration even if its velocity is zero at a given instant of time.  
**Reason :** A body is momentarily at rest when it reverses its direction of motion.  
(1) A (2) B (3) C (4) D
2. **Assertion :** If the displacement of the body is zero, the distance covered by it may not be zero.  
**Reason :** Displacement depends only on end points ; distance (path length) depends on the actual path.  
(1) A (2) B (3) C (4) D
3. **Assertion :** The magnitude of average velocity of an object over an interval of time is either smaller than or equal to the average speed of the object over the same interval.  
**Reason :** Path length (distance) is either equal to or greater than the magnitude of displacement.  
(1) A (2) B (3) C (4) D
4. **Assertion :** An object can have constant speed but variable velocity.  
**Reason :** Speed is a scalar but velocity is a vector quantity.  
(1) A (2) B (3) C (4) D
5. **Assertion :** The speed of a body can be negative.  
**Reason :** If the body is moving in the opposite direction of positive motion, then its speed is negative.  
(1) A (2) B (3) C (4) D
6. **Assertion :** A positive acceleration can be associated with 'slowing down' of the body.  
**Reason :** The origin and the positive direction of an axis are a matter of choice.  
(1) A (2) B (3) C (4) D
7. **Assertion :** The distance traversed, during equal intervals of time, by a body falling from rest are in ratio 1 : 3 : 5 : 7.....[Galileo's law of odd numbers]  
**Reason :** A particle in one-dimensional motion with zero speed may have non-zero velocity.  
(1) A (2) B (3) C (4) D
8. **Assertion :** When a particle moves with constant velocity its average velocity, its instantaneous velocity and its speed are all equal in magnitude.  
**Reason :** If the average velocity of a particle moving on a straight line is zero in a time interval, then it is possible that the instantaneous velocity is never zero in the interval.  
(1) A (2) B (3) C (4) D
9. **Assertion :** In successive time intervals, if the average velocities of a particle are equal then the particle must be moving with constant velocity.  
**Reason :** When a particle moves with uniform velocity, its displacement may increase or decrease with time.  
(1) A (2) B (3) C (4) D
10. **Assertion :** If a body is dropped from the top of a tower of height  $h$  and another body is thrown up simultaneously with velocity  $u$  from the foot of the tower, then both of them would meet after a time  $\frac{h}{u}$ .  
**Reason :** For a body projected upwards, the distance covered by the body in the last second of its upward journey is always 4.9 m irrespective of velocity of projection. ( $g = 9.8 \text{ m/s}^2$ ).  
(1) A (2) B (3) C (4) D
11. **Assertion :** For a moving object,  $|\text{displacement}| \leq \text{distance}$ .  
**Reason :** Instantaneous speed is equal to the magnitude of the instantaneous velocity.  
(1) A (2) B (3) C (4) D



12. **Assertion :** In a free fall, the initial velocity of a body may not be zero.

**Reason :** Free fall means the vertical acceleration of the body is equal to  $g$ .

(1) A (2) B (3) C (4) D

13. **Assertion :** Distance is the actual length of the path but magnitude of displacement is the shortest distance between initial and final positions.

**Reason :** Distance is a scalar quantity and it is always positive or zero but displacement is a vector quantity. It may be positive, negative or zero.

(1) A (2) B (3) C (4) D

14. **Assertion :** Rest (of a body) is a relative term.

**Reason :** Motion of a body may be absolute term.

(1) A (2) B (3) C (4) D

15. **Assertion :** A body moving with constant acceleration, always travels equal distance in equal time intervals.

**Reason :** Motion of the body with constant acceleration is a uniform motion.

(1) A (2) B (3) C (4) D

16. **Assertion :** To cross the river in minimum time, swimmer should swim in a direction perpendicular to the water current.

**Reason :** In this case river flow helps to cross the river.

(1) A (2) B (3) C (4) D

17. **Assertion :** A body dropped from a given height and another body projected horizontally from the same height strike the ground simultaneously.

**Reason :** Horizontal velocity has no effect in the vertical direction.

(1) A (2) B (3) C (4) D

18. **Assertion :** A projectile is thrown with an initial velocity of  $(a\hat{i} + b\hat{j})$  m/s. If range of projectile is maximum then  $a = b$ .

**Reason :** In projectile motion, angle of projection is equal to  $45^\circ$  for maximum range condition.

(1) A (2) B (3) C (4) D

19. **Assertion :** If the position vector of a particle moving in space is given by  $\vec{r} = 2t\hat{i} - 4t^2\hat{j}$ , then the particle moves along a parabolic trajectory.

**Reason :**  $\vec{r} = x\hat{i} + y\hat{j}$  and  $\vec{r} = 2t\hat{i} - 4t^2\hat{j} \Rightarrow y = -x^2$ .

(1) A (2) B (3) C (4) D

20. **Assertion :** In projectile motion, when horizontal range is  $n$  times the maximum height, the angle of projection is given by  $\tan\theta = \frac{4}{n}$

**Reason :** In the case of horizontal projection the magnitude of vertical velocity increases with time.

(1) A (2) B (3) C (4) D

21. **Assertion:** In a projectile motion, the acceleration is constant in both magnitude and direction but the velocity changes in both magnitude and direction.

**Reason :** When a force or acceleration is acting in an oblique direction to the direction of velocity, then both magnitude and direction of the velocity may be changed.

(1) A (2) B (3) C (4) D

22. **Assertion :** In a projectile motion the projectile hits the ground with the same velocity with which it was thrown.

**Reason :** In a projectile motion horizontal velocity remains same but vertical velocity continuously changes and particle strikes the ground with same vertical velocity with which the particle was thrown in vertical direction.

(1) A (2) B (3) C (4) D

23. **Assertion :** In a projectile motion, the vertical velocity of the particle is continuously decreased during its ascending motion.

**Reason :** In projectile motion downward constant acceleration is present in vertical direction.

(1) A (2) B (3) C (4) D

24. **Assertion:** The path of one projectile as seen from another projectile is a straight line.

**Reason :** Two projectiles projected with same speed at angles  $\alpha$  and  $(90^\circ - \alpha)$  have same range.

(1) A (2) B (3) C (4) D

25. **Assertion:** In the projectile motion projected body behaves just like a freely falling body.

**Reason :** There is no change in linear momentum ( $\vec{p} = m\vec{v}$ ) in projectile motion.

(1) A (2) B (3) C (4) D

26. **Assertion :** Projectile motion is uniformly accelerated motion. ( Neglect the effect of air. )

**Reason :** In projectile motion, speed remains constant.

(1) A (2) B (3) C (4) D



27. **Assertion:** Horizontal range is same for angle of projection  $\theta$  and  $(90^\circ - \theta)$ , if speed of projection is same.

**Reason :** Horizontal range is independent of angle of projection.

(1) A (2) B (3) C (4) D

28. **Assertion:** The trajectory of an object moving under constant acceleration due to gravity can be a straight line or parabola.

**Reason :** Initial conditions affect the motion of the object.

(1) A (2) B (3) C (4) D

29. **Assertion:** In order to hit a target, a man should point his rifle at a point higher than the target.

**Reason :** The bullet suffers a vertically downward deflection due to gravity.

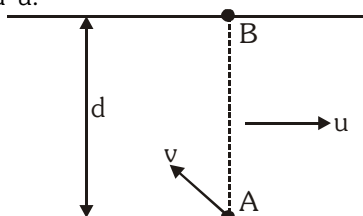
(1) A (2) B (3) C (4) D

30. **Assertion:** Two projectiles of masses  $m$  and  $4m$  when projected with same initial velocity vector have different ranges.

**Reason :** The horizontal range of a projectile depends on mass of the body.

(1) A (2) B (3) C (4) D

31. A man who can swim at a speed  $v$  relative to water wants to cross the river of width  $d$ , flowing with speed  $u$ .



**Assertion:** He cannot reach the point B if  $u > v$ .

**Reason :** The time of crossing is  $\frac{d}{\sqrt{v^2 - u^2}}$  if  $u < v$ .

(1) A (2) B (3) C (4) D

32. **Assertion:** If two particles, moving with constant velocities are to meet, the relative velocity must be along the line joining the two particles.

**Reason :** Relative velocity means motion of one particle as viewed by the other.

(1) A (2) B (3) C (4) D

33. **Assertion:** Two particles start moving with velocities  $\vec{v}_1$  and  $\vec{v}_2$  respectively in a plane. They can meet only if component of their velocities perpendicular to line joining them are equal.

**Reason :** Relative velocity of a body w.r.t. another body is calculated along the line joining them.

(1) A (2) B (3) C (4) D

34. **Assertion:** A coin is allowed to fall in a train moving with constant velocity. Its trajectory is a straight line as seen by an observer attached to the train.

**Reason :** An observer on ground will see the path of coin as a parabola.

(1) A (2) B (3) C (4) D

35. **Assertion:** A man can cross river of width  $d$  in minimum time. On increasing the river velocity, minimum time to cross the river by man will remain unchanged.

**Reason :** As velocity of a river is perpendicular to its width so time to cross the river is independent of velocity of river.

(1) A (2) B (3) C (4) D

## EXERCISE-II (Assertion & Reason)

## ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	1	1	4	1	3	3	4	2	2	1	2	3	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	1	1	1	2	1	4	1	2	3	3	3	1	1	4
Que.	31	32	33	34	35										
Ans.	2	1	3	2	1										

